Mathematics

for the

Senior Phase

Grade 7 – Grade 9

Learner Booklet

(including questions)

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Acknowledgements

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This learner book is not intended to replace the learner’s textbook in the classroom.

Contents

Unit 1.1: The Number System

Real numbers $\mathbb{R}$ are the set of all rational numbers *and* all irrational numbers.

Rational numbers $\mathbb{Q}$

Numbers that can be written as $\frac{a}{b}$ where a and b are integers, $b \neq 0$

Rational numbers *can* be written as a fraction. Both numbers of the fraction must be integers.

- Fractions like $\frac{3}{4}$, $\frac{1}{5}$, $\frac{35}{9}$ and $17\frac{1}{11}$ are all rational numbers.
- Some rational numbers don't look like fractions, but you can change their form:

  $-23 = \frac{-23}{1}$ $\quad -2{,}5 = -2\frac{1}{2} = \frac{-5}{2}$

  $+\sqrt{9} = 3 = \frac{3}{1}$ $\quad 0{,}75 = \frac{3}{4}$

  $0{,}222 \ldots = \frac{2}{9}$ $\quad 347 = \frac{347}{1}$
- All integers are also rational numbers:

  $-3 = \frac{-3}{1}$ $\quad 0 = \frac{0}{1}$ $\quad 3 = \frac{3}{1}$

Integers, whole numbers and natural numbers are all rational numbers.

Integers $\mathbb{Z}$: … –3; –2; –1; 0; 1; 2; 3 …

Whole numbers: 0; 1; 2; 3; 4; …

Natural numbers $\mathbb{N}$: 1; 2; 3; 4; …

Irrational $\mathbb{Q}'$

Numbers that can't be written as fractions with integers in the denominator and the numerator.

The decimal places continue infinitely with no repeating patterns.

Examples:

$\pi \quad \sqrt{3} \quad -\sqrt{17} \quad \sqrt{2}$

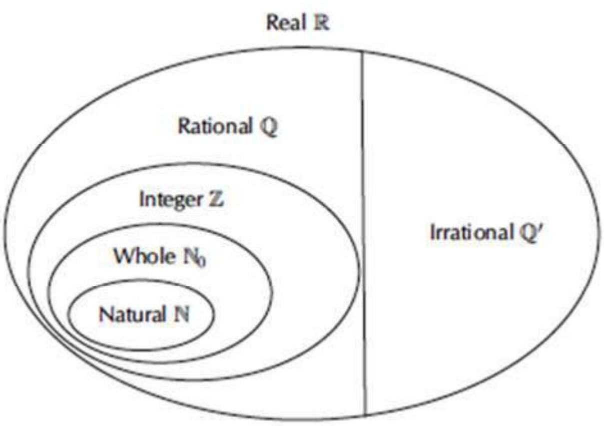
$\sqrt{2} = 1{,}4142413562 \ldots$

$\pi = 3{,}141592654\ldots$

Note that π is irrational, but 3,14 is a rational number very close to π.

Grade 9

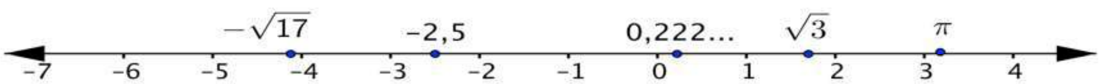
We can represent the different types of real numbers like this:



The real numbers can also be represented on the real number line:

Examples:

We can use approximate values to put irrational numbers like $-\sqrt{17}$; –2,5; 0,222…; π and $\sqrt{3}$ on the number line.



Questions

Unit 1.1: The Number System

Grade 9

1. From each list of numbers, write down those that are rational numbers:

a)	–30,125	$\sqrt{\frac{36}{64}}$	π	34%	45,777…
b)	$\sqrt{0{,}01}$	$\frac{\pi}{3}$	0,333…	$-\sqrt{3}$	4^3
c)	$-\frac{1}{2}$	$\frac{\sqrt{2}}{3}$	$4{,}325325\dot{3}2\dot{5}$		5,23168238
d)	$\frac{22}{7}$	0,00005	–12	$\sqrt{1{,}44}$	

2. Write down 5 numbers which are
   a) negative and rational
   b) positive and rational, but *not* integers
   c) negative and irrational.

3. a) Write down any rational number that lies between 0,1 and 0,2.
   b) Write down a rational number that lies between 0,36 and 0,37.
   c) Write down a rational number that lies between $\frac{1}{3}$ and $\frac{1}{4}$.

4. How many rational numbers are there between $\frac{1}{4}$ and $\frac{1}{2}$?

5. Give three different rational numbers that lie between 0,1 and 0,2.

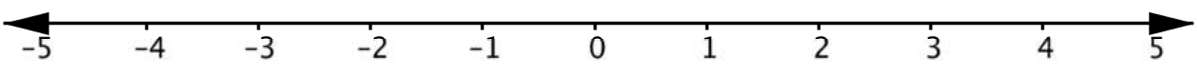
6. a) What integer is closest to 1,3?
   b) Write down a rational number that is less than 1,3 but close to 1,3 in value.
   c) Write down a rational number that is less than 1,3 but even closer to 1,3 than your answer in (b).
   d) Can you find a rational number less than 1,3 that is closest to 1,3?

7. a) Is π exactly equal to $\frac{22}{7}$? Explain.
   b) What integer is closest to π?

8. Decide whether each of the following statements is true or false and give a reason for your answer.
   a) $\sqrt[3]{-1} \in \mathbb{Z}$
   b) $0 \in \mathbb{Q}$
   c) If $a \in \mathbb{Z}$ then $a \in \mathbb{Q}$
   d) If $a \in \mathbb{Z}$ then $a \in \mathbb{N}$

Note: $\in$ stands for "belongs to the set of" so $\sqrt[3]{-1} \in \mathbb{Z}$ is the same as saying $\sqrt[3]{-1}$ is an integer.

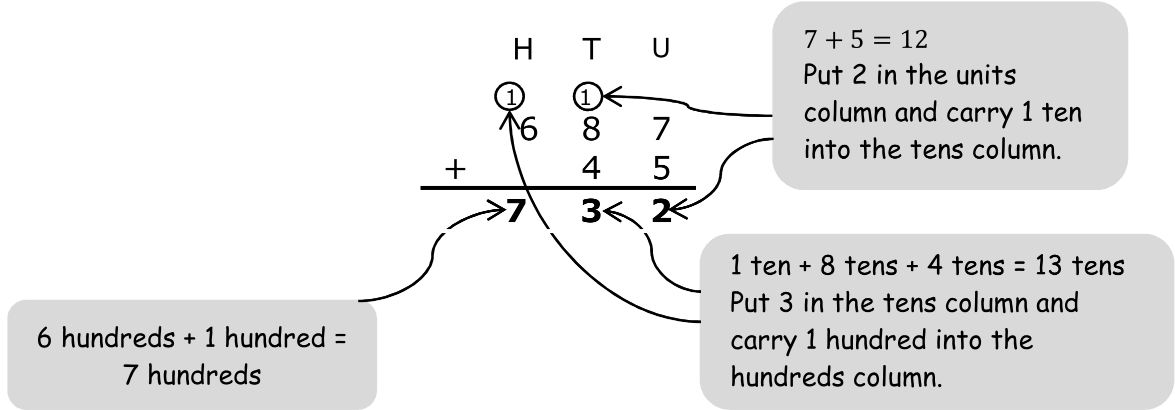
9. Place each of these numbers on the number line below:
   a) $\sqrt[3]{-8}$; 0,3473256; $\frac{\pi}{2}$; $-\frac{5}{3}$; $\sqrt{2}$



Unit 1.2: Operations on whole numbers

A method for adding

Worked example:
Calculate 687 + 45. Put the numbers under each other in columns for hundreds, tens and units.

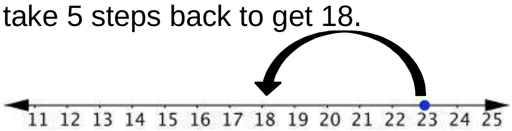


Subtraction

Subtraction as *take away*

Example: 23 – 5 = 18

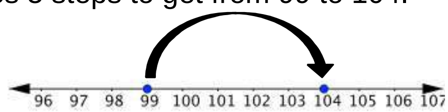
Take 5 away from 23. Start at 23 and take 5 steps back to get 18.



23 – 5 = 18

Subtraction as *difference between*

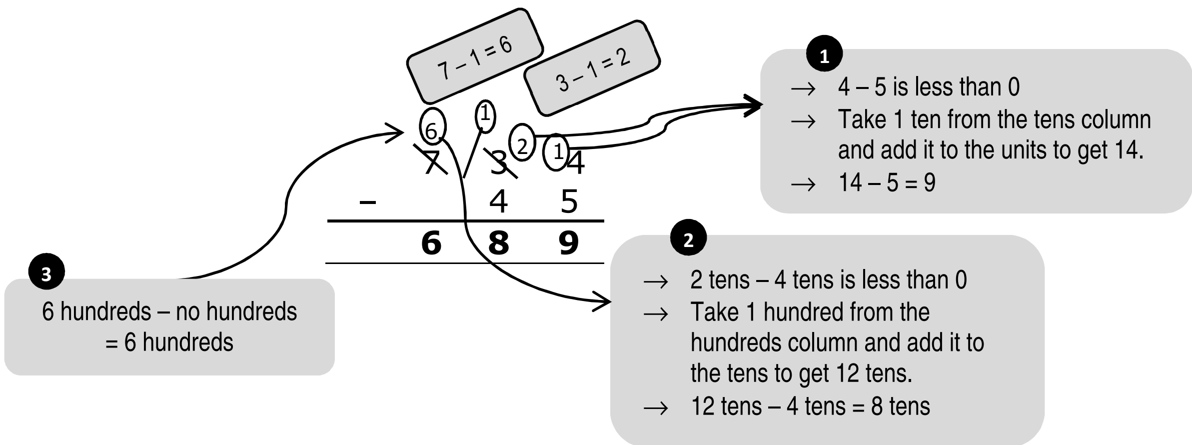
Example: 104 – 99
We find the difference between 99 and 104.
It takes 5 steps to get from 99 to 104.



104 – 99 = 6

A method for subtracting

Worked example: Calculate 734 – 45
Put the numbers under each other in the hundreds, tens and units columns.



A method for multiplying

Worked example: Calculate 13×54

$13 \times 54 = 13(50 + 4)$ so we can calculate 13×50 and 13×4 and then add them.
Put the numbers under each other in the tens and units columns.

$$
\begin{array}{rrr}
 & 1 & 3 \\
\times & 5 & 4 \\
\hline
 & 5 & 2 \\
6 & 5 & 0 \\
\hline
\mathbf{7} & \mathbf{0} & \mathbf{2}
\end{array}
$$

1 To calculate 13×4
→ $3 \times 4 = 12$
So put 2 in the units column and carry 1 to the tens column
→ $1 \times 4 = 4$ and $+1 = 5$

2 To calculate 13×50
→ put 0 in the units column and then just multiply by 5
→ $5 \times 3 = 15$ so put 5 in the tens column and carry 1
→ $5 \times 1 = 5$ and $+1 = 6$

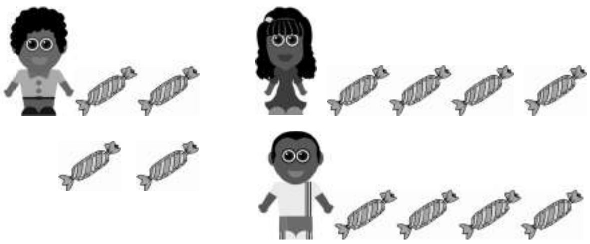
3 Add 52 + 650

Division We can think of division in two ways.

A. Division as making groups:

12 sweets. Put them into packets with 3 in each packet. How many packets?
$12 \div 3$

B. Division as equal sharing:
Share 12 sweets equally between 3 friends. How many sweets will each friend get?



$12 \div 3$

A method for dividing

Worked example: Calculate $192 \div 3$. We want to know how many 3s there are in 192.

$$
\begin{array}{r|rrr}
 & 6 & 4 \\
\hline
3 & 1 & 9 & {}^{1}2
\end{array}
$$

1 There are no 3s in 1 but there are six 3s in 19 with 1 ten remainder.

2 Add the remainder of 1 ten to the units column to get 12.

3 There are four 3's in 12 with no remainder.

Answer:
$$
\begin{array}{r|rrr}
 & 6 & 4 \\
\hline
3 & 1 & 9 & 2
\end{array}
$$

Approximation

It is useful to estimate the answer to a calculation before doing the calculation. It is so easy to make a mistake in a calculation, even when you use a calculator! If your calculated answer is wrong, you will see that it is not close to your estimate.

Worked examples:

1. *Estimate the answer to 48×103.*
   Round off 48 to 50 and 103 to 100.
   $50 \times 100 = 5\ 000$. So $48 \times 103 \approx 5\ 000$.
   If you calculate 48×103, the answer should be close to 5 000.

2. *Approximate the answer to $25\ 116 \div 483$.*
   $25\ 116 \div 483 \approx 25\ 000 \div 500 = 50$. So we expect a calculated answer close to 50.

Questions

Unit 1.2: Operations on whole numbers

No calculators may be used unless otherwise stated.

Grade 7, 8 and 9

1. Calculate:
   a) 238 769 – 141 453 b) 24 502 + 35 798 c) 23 001 – 22 999
   d) 354 231 – 320 000 e) 378 × 45 f) 426 × 243
2. Calculate:
   a) $6\ 251 \div 7$ b) 2 248÷ 4 c) 99 876 × 322
3. A farmer packs 12 apples in every box. If the farmer has 4 272 apples, how many boxes does she need?
4. Mr Mohapi sells your school 500 chairs for R22 500. Mrs Shuma sells your school 200 chairs for R9 600. Who sells cheaper chairs?
5. I have R7 000 to buy T-shirts for camp. If I buy 120 T-shirts at R57 per T-shirt, how much money is left?
6. Vuyo buys bananas from a farm. He buys 50 boxes with 36 bananas in each box and pays R452 in total. He sells the bananas at the market in bags of 3 for R4 a bag. He sells almost all the bananas and has only 18 bananas left at the end of the day.
   How much profit does Vuyo make?
7. You only have R100. There are 5 things you want to buy. They cost R34,50; R24,65; R13,25; R27,50 and R25,70. Do you have enough money to buy all of them?
8. Fill in the blank spaces to make the following number sentences true:
   a) 2 453 + ______ = 3 525 b) 5 ×_______= 3 675
   c) _______ – 9 870 = 4 521 d) ______÷ 3 = 387
   e) 345 – ____ + 123 = 222 f) 7 ×____+ 21 = 343
9. *Estimate* the answer to the following calculations
   a) 2 215 + 3 014 + 5 986 b) 3 962 ÷ 9
   c) 32 × 47 d) 999 × 103
10. Do not actually calculate answers! For each calculation below, decide if the answer is bigger than, smaller than, or equal to 8×27. In each case give a reason for your decision.
    a) 4×27 b) 4×54 c) 24×12 d) $5 \times 8 \times 27 \div 2$
11. Without calculating decide whether each of the calculations below will give an answer that is bigger than, smaller than, or equal to the answer you get when you calculate $3\ 546 - 2\ 397$. In each case give a reason for your decision.
    a) $3\ 546 - 2\ 398$ b) $3\ 547 - 2\ 398$ c) $3\ 547 - 2\ 397$
12. For each of the following calculations an estimate of the answer is given. Without doing the calculations decide whether the estimate is higher than or lower than the actual answer. Give reasons for your answers.
    a) 399× 400 estimate: 160 000
    b) 6 052 ÷ 30 estimate: 200
    c) 80 000 ÷ 380 estimate: 200
13. You are told that $67 \times 63 = 4\ 221$. Use this fact to give the answer to
    a) 67×630 b) $6\ 700 \times 6\ 300$
    c) $4\ 221 \div 63$ d) 68×63

Unit 1.3: Multiples, factors and prime numbers

Grade 7, 8 and 9

- **Multiples:**
  3; 6; 9; 12; 15; 18; 21; 24 … are all positive multiples of 3
  because $3 = 3 \times 1$; $6 = 3 \times 2$; $9 = 3 \times 3$; $12 = 3 \times 4$ etc

- **Common multiples:**
  The multiples of 2 are
  2; 4; 6; 8; 10; 12; 14; 16; 18; 20; …

  The multiples of 3 are
  3; 6; 9; 12; 15; 18; 21; …

  - 6; 12 and 18 are in both lists so they are common multiples of 2 and 3.
  - If we carry on the lists we'll find more common multiples!
  - 6 is the *smallest* multiple that is in both lists and it is called the lowest common multiple.
  - We write **LCM** of 2 and 3 is 6.

- **Factors:**
  1; 2; 3; 4; 6 and 12 are all factors of 12.
  This means that each of these numbers can be multiplied by a whole number to get 12.
  $1 \times 12 = 12$; $2 \times 6 = 12$; $3 \times 4 = 12$
  If you divide 12 by a factor of 12, your answer is a whole number eg $12 \div 4 = 3$

<u>Worked example</u>:

1. Find all the factors of 30.

   Check each number 1; 2; 3; 4; 5; 6; …. Is it a factor of 30?
   - 1 is a factor of 30 since $1 \times 30 = 30$ This means its '*partner*' 30 is also a factor.
   - 2 is a factor of 30 since $2 \times 15 = 30$, This means its '*partner*' 15 is also a factor.
   - 3 is a factor of 30 since $3 \times 10 = 30$, This means its '*partner*' 10 is also a factor
   - 4 is not a factor of 30

   Continue in this way until the factors are repeated:

Factor	Its partner factor
1	30
2	15
3	10
5	6
6	We have listed 6 already, so we can stop looking for factors

   So 1; 2; 3; 5; 6; 10; 15; 30 are the factors of 30

- **Prime numbers:**
  A prime number is a natural number that has exactly 2 factors: 1 and itself. e.g. 23 is a prime number because it has only two factors: 1 and 23.
- **Composite numbers:**
  A whole number that has more than two factors. It has other factors, as well as 1 and itself e.g. 27 is composite because the factors of 27 are 1; 3; 9; 27.
  1 has only one factor so 1 is not prime and 1 is not composite
- **Prime factors:**
  A factor of a number that is prime. For example, the prime factors of 30 are 2; 3 and 5. The other factors of 30 are not prime numbers.

- **Common factors:**
  When a number is a factor of two or more given numbers, it is called a common factor. For example, 3 is a common factor of 30 and 27 because it is a factor of 30 and is also a factor 27.
- **Highest common factor (HCF):**
  The biggest number that is a factor of two or more numbers.

Worked examples:

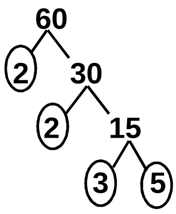
1. Find the highest common factor of 100 and 30.

The factors of 100 are:
1 2 4 5 10 20 25 50 100
The factors of 30 are:
1 2 3 5 6 10 15 30

Circle the factors that are in both lists. 1; 2; 5; and 10 are the common factors of 30 and 100.

10 is the biggest of these common factors. We call it the **highest common factor** of 100 and 30.

2. Write 60 as a product of prime factors.



Split 60 into any two factors. Here we have said 60 = 2 x 30. Circle any primes.
Split any numbers that are not prime into two factors. Here we have said 30 = 2 x 15.
Circle any primes.
Split any numbers that are not prime into two factors. Here we have said 15 = 3 x 5.
Circle any primes.
Stop when all numbers at the bottom of the tree are circled i.e. are prime

So I can say that $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

Grade 7, 8 and 9

- **Using prime factorisation**
  We can use prime factors to find the LCM and HCF of two numbers.

Worked example:
Use prime factors to find the **LCM and HCF** of 60 and 54.

Prime factors of $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$
Prime factors of $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$ *(check you can do this!)*

The **LCM** of 60 and 54 must be a multiple of 54 ***and*** a multiple of 60. So it must contain ***all*** the prime factors that are in 60 and ***all*** the prime factors that are in 54. So look at the prime factors of 60 and 54 and take the **highest power** of each prime factor: The LCM of 60 and 54 will be $2^2 \times 3^3 \times 5 = 4 \times 9 \times 5 = 180$

The **HCF** of 60 and 54 must consist of the factors that are in 60 and also in 54. So look at only the prime factors that are in both 60 and 54 (i.e. 2 and 3) and take the **lowest power** of each: The HCF of 60 and 54 will be $2 \times 3 = 6$

Grade 8 & 9

Questions

Unit 1.3: Multiples, factors and primes

Grade 7, 8 and 9

Know the facts!

1. What is a prime number?
2. What is a composite number?
3. Is 1 a prime number?
4. What number is a factor of every number?
5. List the first seven prime numbers.
6. What number is a factor of all even numbers?
7. Look at this list of numbers. 0; 24; 48; 8; 13; 2; 40; 1; 14
   a) Which numbers are factors of 24?
   b) Which numbers are multiples of 24?
   c) Which numbers are prime?
8. Find the answers quickly.
   a) Find all the factors of 12.
   b) Name all the factors of 24.
   c) Find all the factors of 63.
   d) Find all the factors of 49.
   e) Is 87 a prime number?
   f) Find the highest common factor of 16 and 8.
   g) Find a) the highest common factor (HCF) and b) the lowest common multiple (LCM) of 18 and 24.
   h) Find i) the HCF and ii) LCM of 36 and 99.
   i) Find i) the HCF and ii) LCM of 8, 16 and 24.
   j) Find the HCF of 140 and 168.
   k) Write 1 820 as a product of prime factors.
   l) Write 3 510 as a product of prime factors.
   m) What are the prime factors of 315?
   n) Write 1 320 as a product of prime factors.

More thinking questions.

9. Name any three numbers that have exactly 2 factors.
10. Name any three numbers that have exactly 3 factors.
11. Name any three numbers that have exactly 4 factors.
12. Find an example of a number that has exactly 5 factors.
13. Find a pair of numbers that have an HCF of 12.
14. Find a different pair of numbers that have an HCF of 12.
15. List four numbers between 0 and 100 that are common multiples of 3 and 5.
16. List four numbers between 0 and 100 that are common multiples of 7 and 3.

Grade 8 & 9

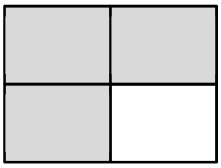
17. Find the highest common factor of 1 820 and 3 510. *(use answers to Q8)*
18. Find the lowest common multiple of 1 820 and 3 510. *(use answers to Q8)*
19. Find the highest common factor of 315 and 1 320. *(use answers to Q8)*
20. Find the lowest common multiple of 315 and 1 320. *(use answers to Q8)*
21. $420 = 2^2 \times 3 \times 5 \times 7$
    Write down a pair of numbers that have an LCM of 420.
22. $420 = 2^2 \times 3 \times 5 \times 7$
    Find a *different* pair of numbers that have an LCM of 420.
23. a is a factor of b
    a) What is the LCM of a and b?
    b) What is the HCF of a and b?
24. If a and b are prime numbers, what is
    a) the LCM and
    b) the HCF of a and b?

Unit 1.4: Fractions

How fractions work

numerator (top number) → 3

denominator (bottom number) → 4



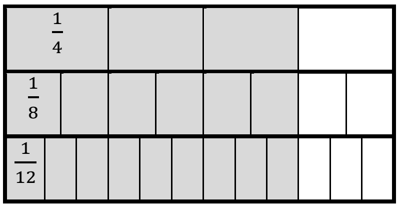
- → 3 pieces of the whole are shaded.
- → There are 4 pieces in the whole altogether.
- → So 3 out of 4 or $\frac{3}{4}$ of the whole is shaded.

Equivalent fractions

Equivalent fractions have the same value so they are equal.

Examples:

a) $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$



b) We can make equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

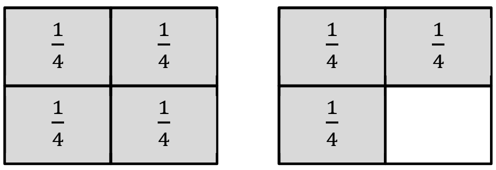
$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

Improper fractions and mixed fractions

An **improper fraction** has a numerator that is larger than its denominator e.g. $\frac{7}{4}$.
$\frac{7}{4}$ is seven quarters.
There are seven $\frac{1}{4}$ pieces.

A **mixed number** is made up of a whole number and a fraction e.g. $1\frac{3}{4}$
$1\frac{3}{4}$ is one and three quarters.
There is 1 whole and three $\frac{1}{4}$ pieces.



one whole

So $\frac{7}{4} = 1\frac{3}{4}$

Worked examples:

1. Write $\frac{7}{4}$ as a mixed number.

$$\frac{7}{4} = 1\frac{3}{4}$$

7 ÷ 4 is 1 whole (4 goes into 7 one time) with 3 quarters left over.

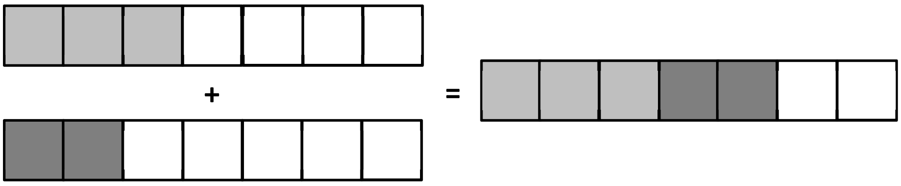
2. Write $3\frac{1}{5}$ as an improper fraction

$$3\frac{1}{5} = \left(3 \times \frac{5}{5}\right) + \frac{1}{5} = \frac{16}{5}$$

Adding and subtracting fractions

If fractions have the same denominator, it is easy to add or subtract them.

Example: $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$



Adding or subtracting fractions with different denominators:

Worked example 1: Calculate $\frac{2}{9}+\frac{5}{12}$

Multiples of 9: 9; 18; 27; 36; 45; 54; …

Multiples of 12: 12; 24; 36; 48; …

So lowest common multiple of 9 and 12 is 36.

$\frac{2 \quad \times 4}{9 \quad \times 4}=\frac{8}{36}$ $\qquad$ $\frac{5 \quad \times 3}{12 \quad \times 3}=\frac{15}{36}$

$\frac{2}{9}+\frac{5}{12}=\frac{8}{36}+\frac{15}{36}=\frac{23}{36}$

There is no number that divides into 23 and 36

So $\frac{23}{36}$ is the answer in simplest form.

1. Find the lowest common denominator.
2. Rewrite each fraction using the lowest common denominator.
3. Add or subtract the fractions.
4. Simplify the answer if necessary.

Worked example 2: Calculate $2\frac{2}{3}-1\frac{1}{6}$

$2\frac{2}{3}-1\frac{1}{6}=\frac{8}{3}-\frac{7}{6}$

Multiples of 3: 3; 6; 9; 12; …

Multiples of 6: 6; 12; 18; …

So lowest common multiple of 3 and 6 is 6

$\frac{8 \quad \times 2}{3 \quad \times 2}=\frac{16}{6}$

$\frac{7}{6}$ already has right denominator

$\frac{8}{3}-\frac{7}{6}=\frac{16}{6}-\frac{7}{6}=\frac{9}{6}$

$\frac{9 \quad \div 3}{6 \quad \div 3}=\frac{3}{2}$

1. Rewrite mixed numbers as improper fractions.
2. Find the lowest common denominator.
3. Rewrite each fraction using the lowest common denominator.
4. Add or subtract the fractions
5. Simplify the answer if necessary.

Grade 7, 8 and 9

Multiplying fractions

Grade 7, 8 and 9

To multiply fractions you multiply the numerators together and multiply the denominators together.

<u>Worked example 1</u>: Calculate $\frac{3}{5} \times \frac{10}{11}$

$\frac{3}{5} \times \frac{10}{11} = \frac{3 \times 10}{5 \times 11} = \frac{30}{55}$

Use common factors to simplify the answer:

$\frac{30}{55} = \frac{30 \div 5}{55 \div 5} = \frac{6}{11}$

<u>Short way</u>:

$\frac{3}{5} \times \frac{10}{11} = \frac{3 \times \cancel{10}^{2}}{{}_{1}\cancel{5} \times 11} = \frac{6}{11}$

It is easier to divide top and bottom by common factors (cancel) before you multiply.

<u>Worked example 2</u>: Calculate $2\frac{1}{3} \times 1\frac{1}{14}$

$2\frac{1}{3} \times 1\frac{1}{14} = \frac{7}{3} \times \frac{15}{14}$

Convert mixed numbers to improper fractions.

$= \frac{{}^{1}\cancel{7} \times \cancel{15}^{5}}{{}_{1}\cancel{3} \times \cancel{14}_{2}}$

Divide by common factors and multiply.

$= \frac{5}{2}$

<u>Worked example 3</u>: Calculate $\frac{2}{3}$ of 12

$\frac{2}{3}$ of 12 means $\frac{2}{3} \times 12$

$\frac{2}{3} \times 12 = \frac{2}{3} \times \frac{12}{1}$

$= \frac{2 \times \cancel{12}^{4}}{{}_{1}\cancel{3} \times 1} = \frac{8}{1} = 8$

Any whole number can be written as a fraction with denominator of 1 e.g. $12 = \frac{12}{1}$

Dividing fractions

Grade 8 and 9

Dividing is the same as multiplying by the inverse. So to divide a fraction, turn the fraction *you are dividing by* upside down and multiply.

<u>Worked example 1</u>: Determine $\frac{3}{4} \div \frac{5}{7}$

$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5}$

$= \frac{21}{20}$

Turn $\frac{5}{7}$ upside down and multiply.

<u>Worked example 2</u>:

$5\frac{1}{3} \div 1\frac{1}{9}$

$= \frac{16}{3} \div \frac{10}{9}$

Convert mixed numbers to improper fractions.

$= \frac{16}{3} \times \frac{9}{10}$

Turn $\frac{10}{9}$ upside down and multiply.

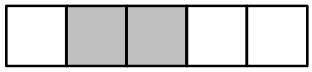
$= \frac{24}{5}$

Questions

Unit 1.4: Fractions

Grade 7, 8 and 9

Routine Questions:



1. What fraction of the rectangle is shaded? Write your answer in the simplest form.
2. If I share 1 chocolate between 7 children fairly, what fraction of the bar will each child get?
3. If I share 3 chocolates between 8 children fairly, what fraction of the bar will each child get?
4. Shade $\frac{1}{5}$ of the squares on the number line below.

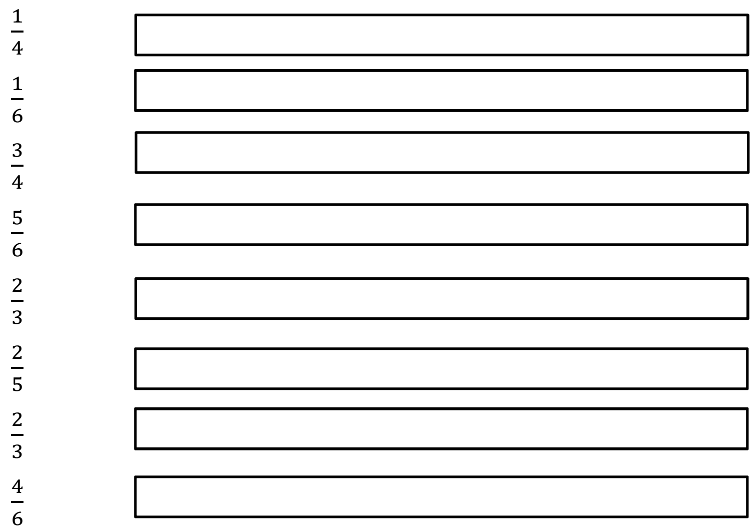
5. Represent the fractions on the diagrams given. Use them to decide which fraction is bigger and then fill in <, > or = between the fractions:

a) $\frac{1}{4}\ \square\ \frac{1}{6}$

b) $\frac{3}{4}\ \square\ \frac{5}{6}$

c) $\frac{2}{3}\ \square\ \frac{2}{5}$

d) $\frac{2}{3}\ \square\ \frac{4}{6}$



6. Fill in the missing numbers

a) $\frac{3}{5}=\frac{\square}{15}$ b) $\frac{2}{7}=\frac{6}{\square}$ c) $\frac{24}{28}=\frac{6}{\square}$ d) $\frac{25}{30}=\frac{\square}{6}$

7. Write the improper fractions as mixed numbers:

a) $\frac{23}{5}$ b) $\frac{36}{11}$ c) $\frac{8}{3}$ d) $\frac{12}{4}$

8. Write the mixed numbers as improper fractions:

a) $5\frac{1}{3}$ b) $4\frac{6}{11}$ c) $7\frac{2}{3}$ d) $6\frac{3}{4}$

9. Calculate and write your answer as a fraction in simplest form:

a) $\frac{3}{8}+\frac{5}{8}$ b) $\frac{7}{9}-\frac{2}{9}$ c) $4\frac{2}{5}+3\frac{1}{5}$ d) $6\frac{1}{3}-2\frac{2}{3}$

e) $\frac{5}{6}+\frac{1}{4}$ f) $\frac{2}{9}+\frac{3}{5}$ g) $\frac{17}{24}-\frac{1}{6}$ h) $3\frac{3}{5}+2\frac{1}{6}$

i) $2\frac{1}{3}-1\frac{2}{4}$ j) $\frac{1}{6}\times\frac{2}{9}$ k) $\frac{2}{11}\times\frac{33}{46}$ l) $\frac{16}{21}\times\frac{3}{4}$

m) $1\frac{3}{7}\times2\frac{1}{3}$ n) $3\frac{2}{5}\times1\frac{1}{4}$ o) $\frac{2}{3}\times\left(1+\frac{1}{2}\right)$ p) $\frac{3}{4}\times\frac{4}{5}+1\frac{2}{5}$

Grade 7, 8 and 9

Word problems and more thinking questions

10. My aunt gave me 10 fizzers. I ate $2\frac{1}{2}$ fizzers on Monday and $3\frac{2}{3}$ fizzers on Tuesday. How many fizzers do I have left?
11. The picture shows $\frac{1}{4}$ of a chocolate bar. Draw a picture of what the whole chocolate bar could look like.

12. The number 60 is

    a) $\frac{1}{4}$ of ______   b) _______ of 80

13. What fraction of an hour and a half is 40 minutes?
14. How many one thirds are there in $4\frac{2}{3}$?
15. I have a bucket with 4 litres of water. I use this water to fill bottles that each take ¾ litres of water. How many bottles can I fill?
16. Which of the following is *not* equivalent to $\frac{1}{3}$?

    a) $\frac{3\,987}{11\,961}$ b) $\frac{4\,952}{14\,856}$ c) $\frac{2\,997}{8\,991}$ d) $\frac{5\,996}{21\,879}$ e) $\frac{1\,999}{5\,997}$

17. I have ½ a litre of orange juice. I drink ¼ of that. How much orange juice is left?

Word problems and more thinking questions

18. My brother won ¾ million rand in the lottery. He gave me ½ million rand. What fraction of his winnings did he give me?
19. $\frac{5}{8}$ of a number is 25. What is the number?
20. Meat costs R32 per kilogram. I buy 2½ kilograms. How much must I pay?
21. My recipe for a batch of biscuits needs ¾ cup of flour. I want to make 6 batches of biscuits. How many cups of flour do I need?
22. After I spend quarter of my money, I have R60 left. How much money do I spend?
23. Five children share 16 sausages equally. How much sausage will each child get?
24. I need ¾ kg to make a loaf of bread. If I have 10 kg of flour, how many loaves of bread can I make?

Grade 8 and 9 only

25. Calculate and write your answer as a fraction in simplest form:

    a) $6 \div \frac{3}{4}$ b) $3\frac{1}{3} \div \frac{5}{9}$ c) $\frac{5}{8} \div 2$

    d) $2\frac{1}{6} \div 1\frac{1}{3}$ e) $1\frac{4}{5} - \frac{8}{15} \div 1\frac{1}{2}$ f) $\frac{3}{4} \times \frac{4}{7} - \frac{6}{7} \div 3$

Unit 1.5: Decimal fractions

Understanding decimals

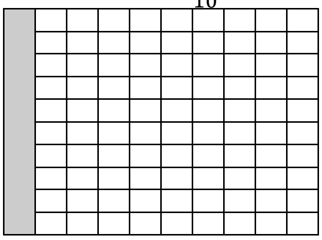
You need to understand place value when you deal with decimal values.
What does 536,247 mean? The digits after the comma represent fractions less than one unit written as tenths, hundredths and thousandths.

hundreds	tens	units	,	tenths	hundredths	thousandths
100s	10s	1s	,	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
5	3	6	,	2	4	7

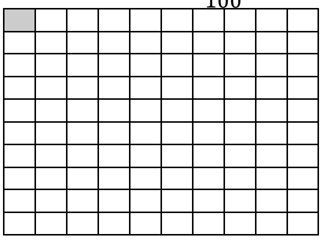
So 536,247 means $5 \times 100 + 3 \times 10 + 6 \times 1 + 2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 7 \times \frac{1}{1\,000}$

Which decimal is bigger? 0,1 or 0,01?

0,1 = $\frac{1}{10}$



0,01 = $\frac{1}{100}$



From the diagrams we see that 0,1 > 0,01
Similarly 0,01 > 0,001
And 0,001 > 0,0001

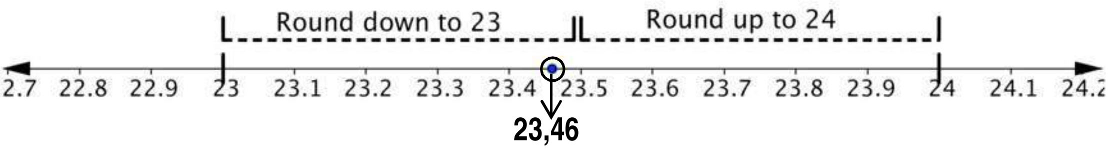
So when we compare decimals, we must look at their place value.
35,**<u>2</u>** is bigger than 35,**<u>0</u>**98 (2 tenths is bigger than no tenths) 35,**<u>2</u>** > 35,**<u>0</u>**98

Rounding decimals

<u>Example 1</u>: Round off 23,46 to the nearest whole number.
Look at the digit to the right of units (whole numbers) in the tenths place of 23,**4**6.
It is 4 which is less than 5 (the half-way point), so we round down to 23.

The closest whole number to the numbers from 23 up to (but not including) 23,5 is 23.

The closest whole number to the numbers from 23,5 to 24 is 24.



<u>Example 2</u>: Round 23,46 to the nearest tenth.
Look at the digit to the right of the tenths i.e. in the hundredth place of 23,4**6**.
It is 6, which is bigger than 5 (the halfway point), so we round up to 23,5.

Converting decimals

a) We can rewrite a decimal as a fraction using our knowledge of place value:
0,62 = $\frac{6}{10} + \frac{2}{100} = \frac{60}{100} + \frac{2}{100} = \frac{62}{100}$. The fraction $\frac{62}{100}$ can be simplified to $\frac{31}{50}$.

b) A fraction can be written as a decimal by writing it with a denominator of 10; 100 or 1 000. So $\frac{1}{4} = \frac{25}{100} = 0{,}25$.

If it is difficult to rewrite a fraction with a denominator of 10; 100 or 1 000, then you can convert it using division.

<u>Worked example</u>: $\frac{1}{8} = 1 \div 8 = \begin{array}{r|l} & 0{,}\ 1\ 2\ 5 \\ \hline 8 & {}^{0}1,\ {}^{1}0\ {}^{2}0\ {}^{4}0 \end{array} = 0{,}125$

You can also work this out using your calculator.

Grade 7, 8 & 9

Adding and subtracting decimals

Add or subtract decimals in the same way as we did for whole numbers. Line up the numbers carefully according to place value.

15,03 can be written as 15,030

Example 1:

26,473 + 18,25 = 44,723

$$
\begin{array}{rcccccc}
 & ^{1}2 & 6 & , & ^{1}4 & 7 & 3 \\
+ & 1 & 8 & , & 2 & 5 & 0 \\
\hline
 & 4 & 4 & , & 7 & 2 & 3 \\
\hline
\end{array}
$$

Example 2:

15,03 – 2,751 = 12,279

$$
\begin{array}{rcccccc}
 & 1 & ^{4}\cancel{5} & , & ^{9}\cancel{^{1}0} & ^{12}\cancel{3} & ^{1}0 \\
- & & 2 & , & 7 & 5 & 1 \\
\hline
 & 1 & 2 & , & 2 & 7 & 9 \\
\hline
\end{array}
$$

We 'carry' as we did for whole numbers

Align the place value columns by lining up the commas.

We 'borrow' as we did for whole numbers

Multiplying and dividing decimals by 10, 100 or 1 000

Our number system is based on the number 10, so multiplying or dividing by 10, 100 or 1 000 just shifts the place value.

- Each time you multiply by a 10, the digits shift up one decimal place.
  Moving the comma one place to the right will shift the digits up one decimal place.
- Each time you divide by a 10, the digits shift down by one decimal place.
  Moving the comma one place to the left will shift the digits up one decimal place.

Examples:

$2{,}03 \times 10 = 20{,}3$
$34{,}12 \div 10 = 3{,}412$

For 10, move one place.

$531{,}289 \times 100 = 53128{,}9$
$531{,}289 \div 100 = 5{,}31289$

For 100, move 2 places.

Multiplying decimals

Multiply each decimal by a power of 10, to get rid of the comma. After calculating the answer, correct for this and divide by the same power of 10.

Example: $0{,}02 \times 7{,}4$

$$
\begin{array}{ccc}
0{,}02 & \times & 7{,}4 \\
\downarrow & & \downarrow \\
\boxed{\times 100} & & \boxed{\times 10} \\
\downarrow & & \downarrow \\
2 & \times & 74
\end{array}
= 148 \rightarrow \boxed{\div 100} \rightarrow \boxed{\div 10} \rightarrow \mathbf{0{,}148}
$$

Multiply 0,02 by 100 and 7,4 by 10 to get rid of the decimals.

Divide 148 by 100 and by 10 to make the final answer correct

We can summarise what we have done as follows

Multiply $2 \times 74 = 148$ and so $\underbrace{0{,}02}_{\text{2 places}} \times \underbrace{7{,}4}_{\text{1 place}} = \underbrace{0{,}148}_{\text{3 places}}$

Dividing decimals

- First write the division of decimals as a fraction.
- Then multiply top and bottom by a power of 10 – choose these to make the denominator a whole number.

Example:

$$
\begin{aligned}
& 6{,}51 \div 0{,}3 \\
&= \frac{6{,}51}{0{,}3} \\
&= \frac{6{,}51 \times 10}{0{,}3 \times 10} = \frac{65{,}1}{3} = 21{,}7
\end{aligned}
$$

Multiply numerator and denominator by the same number to make denominator a whole number.

Grade 7, 8 & 9

Questions

Unit 1.5: Decimal numbers

Grade 7, 8 and 9

1. Write each of these decimal numbers as fractions in their simplest form.

a) 0,8 b) 0,02 c) 0,005 d) 0,25 e) 0,305
f) 4,05 g) 80,75 h) 3,12 i) 54,89 j) 10,379

2. Write each of these fractions as decimals:

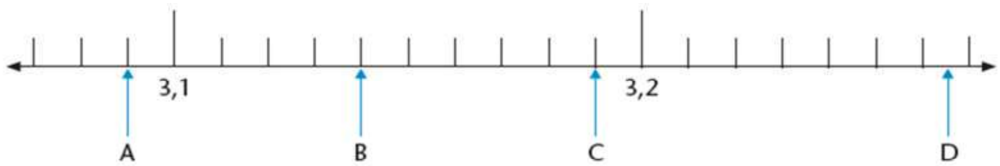
a) $\frac{2}{10}$ b) $\frac{7}{100}$ c) $\frac{9}{1\,000}$ d) $\frac{43}{100}$ e) $\frac{231}{1\,000}$
f) $\frac{34}{1\,000}$ g) $\frac{1}{5}$ h) $\frac{3}{4}$ i) $3\frac{14}{100}$ j) $10\frac{1}{2}$

3. Write the following in ascending order (i.e. from smallest to biggest):

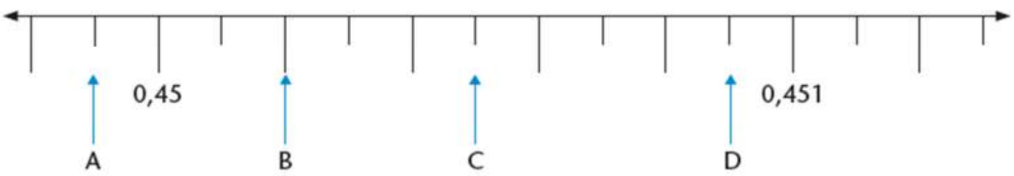
a) 0,3; 0,03; 0,042 b) 0,2; 0,21; 0,12 c) 0,7; 0,3256; 0,09
d) 0,98; 1; 0,0643 e) 0,78; 0,6; 0,09 f) 0,0045; 0,006; 0,01

4. Write down the numbers at points A; B; C; and D on the number lines.

a)



b)



5. a) Round off 6 543,2348 to the nearest (i) thousand (ii) ten (iii) tenth (iv) thousandth.
b) Round off 7 269,8063 to the nearest (i) hundred (ii) whole number (iii) tenth (iv) hundredth.
c) Round off 6 399,2318 to the nearest (i) thousand (ii) ten (iii) tenth (iv) thousandth.
d) Round off 6 053,7247 to the nearest (i) hundred (ii) ten (iii) tenth (iv) thousandth.
e) Round off 689,9828 to the nearest (i) thousand (ii) ten (iii) tenth (iv) thousandth.

6. Calculate (without using a calculator)

a) 3,3 + 4,83 b) 9,4 + 3,7 c) 3,56 + 4,689 d) 9,43 – 3,21
e) 9,43 – 3,56 f) 0,06 + 3,2 + 5,75 g) 0,07 + 4,21 – 4,2 h) 1 – 0,03
i) 100 – 0,03 j) 0,04 + 5,06

7. Fill in the missing value:

a) 13,3 + 1,4 + ___ = 18,2 b) 25,31 – ___= 15,06 c) 14,9 + ____= 15
d) 14,09 + ____= 15 e) 1 – ____= 0,3 f) 1 – _____= 0,78

8. Calculate (without using a calculator):

a) $51{,}7 \times 100$ b) $0{,}2 \div 10$ c) $0{,}071 \times 1\,000$
d) $521{,}23 \div 100$ e) $31{,}75 \times 10$ f) $0{,}03 \div 10$
g) $1{,}2 \div 100$ h) $2{,}6 \div 10$ i) $5\,413{,}217 \div 1\,000$

9. Calculate (without using a calculator):

a) $4 \times 0{,}3$ b) $0{,}02 \times 0{,}3$ c) $7{,}2 \div 9$ d) $(0{,}1)^2$
e) $451{,}2 \div 0{,}02$ f) $2{,}1 \div 0{,}3$ g) $5{,}365 \div 0{,}05$ h) $1{,}44 \div 1{,}2$
i) $0{,}8 \times 3{,}81$ j) $1{,}2 \times 3{,}4$

10. You are told $45 \times 24 = 1\ 080$. Use this to determine:

a) $4{,}5 \times 2{,}4$ b) $0{,}045 \times 0{,}24$ c) $0{,}45 \times 240$

11. You are told $23 \times 37 = 851$. Use this to determine:

a) $2{,}3 \times 3{,}7$ b) $0{,}23 \times 0{,}37$ c) $0{,}23 \times 370$

12. Calculate (without using a calculator):

a) $0{,}35 + 0{,}2 \times 0{,}1$ b) $0{,}42 \div 0{,}2 + 3{,}1 \times 3$
c) $\frac{3{,}2 + 4{,}05}{0{,}05}$ d) $0{,}75 + 0{,}1 \times 2$
e) $0{,}24 \div 0{,}3 + 1{,}4 \times 2$ f) $\frac{3{,}4 + 4{,}92}{0{,}02}$

13. At a supermarket, beef is sold at R32,60 per kilogram.

a) How much does 2 kg of beef cost?
b) How much does 0,2 kg of beef cost?
c) How much does 0,5 kg of beef cost?

14. A municipality charges R14,06 per kilolitre of water.

a) How much will 5 kilolitres of water cost?
b) How much will 6,5 kilolitres of water cost?

15. Which of these is bigger?
a) 0,04 or $(0{,}04)^2$
b) 0,04 or $\sqrt{0{,}04}$
c) 0,3 or $(0{,}3)^2$
d) 0,3 or $\sqrt{0{,}3}$

16. What is the next number in the pattern?

a) 2,7; 3,0; 3,3; 3,6; 3,9; ____.
b) 3,28; 3,3; 3,32; 3,34; 3,36; 3,38; ____.
c) 0,89; 0,81; 0,73; 0,65; 0,57; ____.
d) 0,8; 1,6; 3,2; 6,4; 12,8; ____.

Unit 1.6: Percentages

$x\% = \frac{x}{100}$ Percent means *'out of 100'*

Converting between fractions, decimals and percentages

Worked examples:

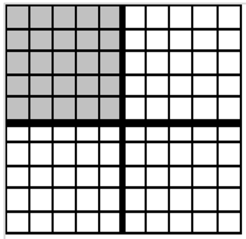
a) Convert 25% to a fraction in simplest form.

$25\% = \frac{25}{100}$

1 Rewrite % as 'out of 100'.

$= \frac{1}{4}$

2 Simplify fraction. Divide top and bottom by the common factor of 25.



b) Convert 0,3 to a percentage.

$0{,}3 = \frac{3}{10}$

1 Rewrite decimal as a fraction.

$= \frac{3}{10} \times \frac{10}{10} = \frac{30}{100}$

2 Make an equivalent fraction with a denominator of 100.

$= 30\%$

3 Rewrite it as a percentage.

Calculating a percentage of an amount

To calculate $x\%$ of an amount, use $\frac{x}{100} \times$ the amount

Worked example: 20% of R75 $= \frac{20}{100} \times \text{R}75$

$= \frac{20}{100} \times \frac{\text{R}75}{1}$

$= \frac{{}^{1}\cancel{20} \times 75}{{}_{5}\cancel{100} \times 1}$

$= \frac{\text{R}75}{5} = \text{R}15$

Finding what percentage a part of a whole is

To find what percentage a part (x) is of a whole (y), we want the fraction $\frac{x}{y}$ of 100%

Worked example:

21 learners out of the 420 learners in a school sing in the choir.
What percentage of the learners sings in the choir?

$\frac{21}{420}$ of 100%

$= \frac{21}{420} \times \frac{100}{1}\%$

$= \frac{{}^{1}\cancel{21}}{{}_{20}\cancel{420}} \times \frac{100}{1}\%$

$= \frac{100}{20}\% = 5\%$

Grade 7, 8 and 9

Calculating percentage increase or decrease

To find the percentage by which something has increased or decreased we calculate

$$= \frac{\text{new amount} - \text{original amount}}{\text{original amount}} \times 100\%$$

Worked example:

The number of learners taking maths drops from 500 to 470.

What percentage of the learners drop maths?

$$\% \text{ decrease } = \frac{\text{new amount} - \text{original amount}}{\text{original amount}} \times 100\%$$

$$= \frac{470 - 500}{500} \times 100\% = -6\%$$

The negative shows that this is a decrease in percentage, not an increase.

Increase or decrease by a percentage

→ To increase an amount by x% , you add x% of the amount to the amount.

$$\text{amount} + x\% \text{ of amount} = \text{amount} + \frac{x}{100} \times \text{amount} = (1 + \frac{x}{100}) \times \text{amount}$$

→ To decrease an amount by x%, you subtract x% of the amount to the amount

$$\text{amount} - x\% \text{ of amount} = \text{amount} - \frac{x}{100} \times \text{amount} = (1 - \frac{x}{100}) \times \text{amount}$$

Worked examples:

1. The price of milk was R11, but is increased by 5%. What is the new price of milk?

New price = R11 + 5% of R11 = R11 + $\frac{5}{100} \times \text{R11}$

$= \left(1 + \frac{5}{100}\right) \times \text{R11} = (1{,}05) \times \text{R11} = \text{R11{,}55}$

If an amount is increased by 5%, you multiply by 1,05. If the amount was increased by 43% you would multiply by 1,43.

2. You get a discount of 25% on a shirt that costs R400. How much do you pay for the shirt?

New price = R400 – 25% of R400 = R400 – $\frac{25}{100} \times \text{R400}$

$= \left(1 - \frac{25}{100}\right) \times \text{R400} = (0{,}75) \times \text{R400} = \text{R300}$

Percentage back to whole

If $x\% = \text{an amount}$ then $1\% = \frac{\text{amount}}{x}$ so $100\% = \frac{\text{amount}}{x} \times 100$

$$\begin{array}{ccc} & x\% = \text{an amount} & \\ \div x \downarrow & & \downarrow \div x \\ & 1\% = \frac{\text{an amount}}{x} & \\ \times 100 \downarrow & & \downarrow \times 100 \\ & 100\% = \frac{\text{an amount}}{x} \times 100 & \end{array}$$

Worked example:

30% of my class is boys. There are 12 boys in my class. How big is my class?

30% = 12 learners 1% = $\frac{12}{30}$ so 100% $= \frac{12}{30} \times 100 = 40$ learners

There are 40 learners in my class.

Grade 7, 8 and 9

Questions

Unit 1.6: Percentage

No calculators may be used unless otherwise stated.

Grade 7, 8 and 9

1. Write each of the following percentages as a fraction (in simplest form) and as a decimal:
   a) 1% b) 2% c) 30% d) 25% e) 75%
   f) 50% g) 45% h) 68% i) 40% j) 55%

2. Write each of the following decimals as percentages
   a) 0,04 b) 0,09 c) 0,6 d) 0,27 e) 0,25
   f) 0,8 g) 0,32 h) 1 i) 0,295 j) 0,075

3. Write each of the following fractions as percentages
   a) ¼ b) ½ c) ¾ d) $\frac{1}{25}$ e) $\frac{2}{25}$
   f) $\frac{3}{20}$ g) $\frac{12}{50}$ h) $\frac{7}{20}$ i) $\frac{3}{5}$ j) $\frac{9}{25}$

4. Write the following in ascending order (i.e. from smallest to biggest)
   a) $\frac{1}{5}$, 15% and 0,25 b) $\frac{1}{25}$, 5% and 0,02 c) $\frac{3}{10}$, 3% and 0,32
   d) $\frac{23}{50}$, 40% and 0,06 e) ½ , 35% and 0,4 f) $\frac{3}{25}$, 0,1 and 15%

5. Calculate:
   a) 10% of 600 b) 25% of 368 c) 75% of 328
   d) 20% of 125 e) 40% of 500 f) 30% of 12 300

6. a) I score 40% for my maths test. If the test is out of 30 marks, what is my mark?
   b) I get 24 out of 30 for my maths test. What percentage do I get for the test?
   c) 25% of the learners at the camp are boys. There are 60 learners at the camp. How many are boys?
   d) I win R1 000 and decide to give 15% of this to my brother. How much money does my brother get?
   e) I took 14 litres from a water tank. This is 20% of the water in the water tank. How much water is left in the water tank?
   f) The price of milk increases from R10 to R11,90. By what percentage does the price of milk increase?
   g) The number of people who attend a clinic monthly drops from 40 to 16. By what percentage does the number of people attending the clinic drop?
   h) A shirt cost R600. I get 20% discount. How much do I pay?
   i) The price of a car is R150 000 without VAT. What is the price when 14% VAT is added?

7. a) I buy a farm for R4 000 000 and sell it for R6 million. What percentage profit do I make?
   b) The price of a bicycle is R3 000. The store owner sells it to me for R2 760. What percentage discount is she giving me?
   c) We collect R3 000 in our fundraising drive. This is only 20% of the total amount we need to collect. What is the total amount we need to collect?
   d) A plumber charges me R400 for a job. He then adds 14% VAT to the bill. How much must I pay him in total? Grade 8 & 9 only
   e) An estate agent earns 3% commission on a house she sells. So she gets paid 3% of the amount she sells the house for. If she earns R10 000 in commission, how much does she sell the house for? Grade 9 only
   f) Thandi buys a book for R300 and sells it for R360. Her brother buys a book for R450 and sells it for R530. Which of them made the greater *percentage* profit?

Unit 1.7: Simple and compound interest

Simple interest

Grade 7, 8 and 9

Simple interest is interest charged (or earned) on the initial amount borrowed (or invested).

Worked example:

If I invest R500 in an account that pays 10% simple interest per year, I will earn 10% of R500 = $\frac{10}{100} \times \text{R}500 = \text{R}50$ as interest each year. So if I invest it for 3 years, I'll earn $3 \times \text{R}50 = \text{R}150$ in interest. So at the end of 3 years, I'll have R650 in total.

We use the formula:

$$\boxed{\text{A} = \text{P}(1 + in)}$$

$$\boxed{\text{SI} = \text{P}.n.i}$$

A = total paid back (loan) or accumulated (investment)
SI = simple interest charged (loan) or earned (investment)
P = amount borrowed (loan) or invested (investment)
i = interest rate
n = number of years

Hire Purchase

Grade 8 and 9

Customer wants to buy something expensive but doesn't have cash to pay for it immediately → Store allows customer to pay a deposit and remaining amount is a loan → Store charges simple interest on loan and calculates how much the customer owes → Customer pays off the loan plus interest in monthly amounts

Worked example:

I buy a fridge on hire purchase for R3 000. I pay a 10% deposit. The store charges 25% interest per year and allows me to pay it back over 2 years. How much do I need to repay each month?

Deposit = 10% of R3 000 = R300
So hire purchase loan = R3 000 – R300 = R2 700
Amount I need to repay including interest:

$\text{A} = \text{P}(1 + in)$ $\quad\quad$ $\text{P} = \text{R}2\,700;\ i = 25\% = 0{,}25;\ n = 2$

$\text{A} = \text{R}2\,700(1 + 0{,}25 \times 2) = \text{R}4\,050$

R4 050 over 24 months is $\frac{\text{R}4050}{24} = \text{R}168{,}75$ per month.

Compound interest

Grade 9

Compound interest is interest charged (or earned) on the initial amount, as well as on any interest already charged (or earned).

We use the formula:

$$\boxed{\text{A} = \text{P}(1 + i)^n}$$

A = total paid back (loan) or accumulated (investment)
P = amount borrowed (loan) or invested (investment)
i = interest rate
n = number of years

Worked example:

I invest R5 000 in the bank for 6 years and they pay me 8% compound interest per year. How much will I have in the bank after 6 years?

$\text{A} = \text{P}(1 + i)^n$ $\quad\quad$ $\text{P} = \text{R}5\,000;\ i = 8\% = 0{,}08;\ n = 6$

$\text{A} = \text{R}5\,000(1 + 0{,}08)^6$

$\quad = \text{R}5\,000(1{,}08)^6 = \text{R}7\,934{,}37$

Questions

Unit 1.7: Simple and compound interest

Grade 7, 8 and 9

1. Calculate the total amount I will have in the bank after 3 years if
   I invest R1 000 at 5% per year a) simple interest b) compound interest
   I invest R5 000 at 10% per year c) simple interest d) compound interest

2. Calculate the total amount of interest I will pay after 3 years if
   a) I borrow R3 000 at 4% per year simple interest
   b) I borrow R3 000 at 4% per year compound interest
   c) I borrow R6 000 at 12% per year simple interest
   d) I borrow R6 000 at 12% per year compound interest.

3. I invested some money in the bank at 5% per year simple interest. After a year I had earned R10 in interest. How much money did I invest?

4. I invested some money in the bank at 10% per year simple interest. After 3 years I had earned R120 in interest. How much money did I invest?

5. I borrow R2 000 and I am charged 5% simple interest per month. If I don't pay the money back, after how many months will I owe double the amount I borrowed?

Grade 8 and 9

6. I buy a TV that costs R4 000 on hire purchase. The store charges me 15% interest per year. I pay back over 3 years.
   a) What is the total amount I need to pay back?
   b) What are my monthly installments?

7. You buy a computer that costs R8 000 on hire purchase. You pay a deposit of 20%. You are charged 25% interest per year on the remaining amount. What are the monthly installments if you pay back the loan over 5 years?

Grade 9

8. Calculate the total amount I will have in the bank after 5 years if
   a) I invest R1 000 at 8% per year compound interest
   b) I invest R8 000 at 8% per year compound interest

9. Calculate the total amount of interest I will pay after 5 years if
   a) I borrow R1 000 at 16% per year compound interest.
   b) I borrow R8 000 at 16% per year compound interest.

10. A money lender charges 10% simple interest per month. I know that in 6 months' time I will get my bonus of R5 000. How much money can I borrow now if I want to pay the loan (with interest) off in full in 6 months' time?

11. I invest R3 000 in a savings account which earns 8% compound interest per year. I leave it in the account and, at the end of 5 years, I add R4 000 to the account. Then I leave the money in the account for another 2 years. How much will I have in my account at the end of another 2 years?

12. Would you prefer to earn 5% simple interest per year, or 5% compound interest per year? Why?

13. You have R3 000 to invest. Which investment will give you more:
    i) a savings account that pays 8% simple interest per year for 5 years
    ii) A savings account that pays 7% compound interest per year for 5 years.

Unit 1.8: Ratio and rate

Ratio

A ratio compares two quantities *of the same kind* (e.g. people, cups, kilometres etc)

Example:

In a recipe I use 3 cups of milk for every 2 cups of flour. This tells us the ratio of milk to flour. We can write the ratio in different ways:

3 to 2 3 **:** 2 $\frac{3}{2}$

Equivalent ratios

Ratios can be written as fractions. Equivalent ratios are equal, but the numbers in the top and bottom of the fractions are different. To find equivalent ratios, we can multiply or divide each number in the ratio by the same amount.

Example:

The ratios $\frac{12}{16}, \frac{3}{4}$ and $\frac{15}{20}$ are equivalent.

$$\begin{array}{rccccl} \div 4 & 12 & : & 16 & \div 4 \\ & 3 & : & 4 & \\ \times 5 & 15 & : & 20 & \times 5 \end{array}$$

Part-part ratios:

Worked example:

At a camp, there is a ratio of children to adults of 5 **:** 2.

If there are 30 children at the camp, how many adults are there?

$$\begin{array}{rcccl} \times 6 & 5 & : & 2 & \times 6 \\ & 30 & : & ? & \end{array}$$

The number of adults is $2 \times 6 = 12$.

Part-whole ratios:

Worked example:

At a camp there is a 5 **:** 2 ratio of children to adults. If there are 140 people at the camp altogether, how many children and how many adults are there?

We know that for every 5 children there are 2 adults.

So there are 5 children in every group of 5 + 2 = 7 people.

children **:** people

$$\begin{array}{rcccl} \times 20 & 5 & : & 7 & \times 20 \\ & ? & : & 140 & \end{array}$$

So the number of children = $5 \times 20 = 100$.

The number of adults = 140 – 100 = 40.

Rate

A rate is similar to a ratio, but a ratio compares two *different* kinds of quantities.

Example:

A car can travel 26 km on 2 litres of petrol. How far will it travel on 6 litres of petrol?

$$\begin{array}{rcccl} \times 3 & 26 \text{ km} & : & 2 \text{ litres} & \times 3 \\ & ? & : & 6 \text{ litres} & \end{array}$$

The distance the car can travel will be $26 \times 3 = 78$ km

Grade 7, 8 and 9

Grade 7, 8 and 9

Using unit ratios and rates

Sometimes it is not easy to see what number to multiply by to get the equivalent ratio we want. It helps to find a unit ratio or rate (a ratio or rate with 1 in it) first and then get the ratio we want.

Worked example:

A factory produces 261 boxes in 3 hours. If the factory produces boxes at that rate, how long will it take them to make 870 boxes?

261 boxes : 3 hours
870 boxes : ? hours

It is not easy to see what we've multiplied by so we will first make a ratio with 1.

The time taken to make 1 box

$$\div 261 \left(\begin{array}{l} 261 \text{ boxes} : 3 \text{ hours} \\ 1 \text{ box} : \frac{3}{261} \text{ hours} \end{array} \right) \div 261$$

Use your calculator.

The time taken to make 870 boxes:

$$\times 870 \left(\begin{array}{l} 1 \text{ box} : \frac{3}{261} \text{ hours} \\ 870 \text{ boxes} : \frac{3}{261} \times 870 \text{ hours} \end{array} \right) \times 870$$

So it will take $\frac{3}{261} \times 870 = 10$ hours

Grade 9

Direct and indirect proportion

Direct proportion – two quantities are *directly proportional* if they have a constant quotient (the answer when you divide).

Worked example:

Distance (km)	60	120	180	210	240
Time (hours)	1	2	3	3,5	4

$\frac{\text{distance}}{\text{time}}$ gives the *constant speed* of 60 km/h.

$\frac{60}{1} = \frac{120}{2} = \frac{180}{3} = \frac{210}{3,5} = \frac{240}{4} = 60$ km/h $\rightarrow$ $\frac{\text{distance}}{\text{time}}$ is constant.

The distance travelled *increases* in *direct proportion* to the time. As the time increases, the distance also increases.

Indirect (inverse) proportion – two quantities are indirectly or inversely proportional if they have a constant product (the answer when you multiply).

Worked example:

It takes 24 days for 1 worker to paint a factory. The table shows the number of days it takes depending on how many workers are employed.

Number of days	24	12	8	4	2
Number of workers	1	2	3	6	12

$24 \times 1 = 12 \times 2 = 8 \times 3 = 4 \times 6 = 2 \times 12 = 24 \rightarrow$ no days $\times$ no workers is constant

The number of days to paint the factory *decreases* as the number of workers employed *increases*.

The number of days and number of workers are *indirectly proportional*.

Questions

Unit 1.8: Ratio and rate

No calculators may be used unless otherwise stated.

Grade 7, 8 and 9

1. To make a cooldrink I must mix concentrate with water in a ratio of 1 **:** 3.
   a) If I have 400 ml of concentrate, how much water must I use?
   b) If I want to make 8 litres of cooldrink, how much concentrate must I use?
   c) If I want to make 1 litre of cooldrink, how much concentrate must I use?
   d) If I want to make 10 litres of cooldrink, how much water must I use?

2. Share R240 between the people in the ratio shown:
   a) Nathi's share **:** Ayanda's share = 1 **:** 5
   b) Thandiswa's share **:** Nathalia's share = 5 **:** 7
   c) Anna's share **:** Tebogo's share **:** Khotso's share = 1 **:** 2 **:** 3

3. If the cost of one US dollar ($) is R10,30:
   a) how many rands will it cost you to get $50?
   b) how many dollars will you get for R1 030?
   c) how many dollars will you get for R100?

4. A recipe to make 30 biscuits uses 2 eggs.
   a) How many eggs will I need to make 120 biscuits?
   b) If I have 7 eggs, how many biscuits can I make?

5. I travel at 80 km per hour. How long will it take me to travel
   a) 400 km? b) 600 km?
   How far will I have travelled in
   c) 4½ hours? d) 12 minutes?

6. If the exchange rate for the British pound to the South African rand is given as £1 = R17,20 calculate:

   a) how many rands you will get for £10?
   b) how many rands you will get for £28?
   c) how many pounds you will get for R100?
   d) how many pounds R1 is worth?

7. The dollar($) / euro(€) exchange rate is given as $1 = €0.80.

   a) How many dollars is one euro worth?
   b) The dollar-rand exchange rate is given as $1 = R12. How many euros will I get for R100?

8. There is enough food in the warehouse to feed one person for 30 days or 2 people for 15 days etc.
   a) How many days will the food last if there are 10 people who need food?
   b) The food must last for 5 days. How many people can we feed?

9. Beef cost R35 per kilogram. What is the cost of:
   a) 4 kg b) ½ kg
   c) 400 g d) 600 g

10. Cooldrink is being sold at the supermarket for the following prices:
R3,50 for a 200 ml can; R4,50 for a 340 ml can; R6,90 for a 500 ml bottle and R12 for a 2 litre bottle.
a) Calculate the cost per litre for each can or bottle.
b) Determine which cooldrink is the cheapest per ml.

11. Andile travelled 532 km in 5 hours, Bongani travelled 392 km in 3 hours and Chris travelled 200 km in 2 hours. Who travelled fastest?

12. A map is drawn with a scale of 1 **:** 30 000. I measure the distance between my house and the school on the map and it is 5 cm. How far is my house from the school in km?

13. I buy 8 cakes for R92. How much will 10 of the same cakes cost?

Grade 9 only

14. For each of the tables below state whether x and y are
i. directly proportional to each other
ii. inversely proportional to each other or
iii. neither (not proportional)

a)

x	3	5	7	9	11
y	9	15	21	27	33

b)

x	2	4	6	8	10
y	18	9	6	4,5	3,6

c)

x	3	4	5	6	7
y	5	7	9	11	13

d)

x	3	4	5	6	7
y	0,6	0,8	1	1,2	1,4

15. Find the values of p and q if
a) x and y are directly proportional to each other
b) x and y are inversely proportional to each other

x	5	12,5	q
y	10	p	2

Unit 1.9: Integers

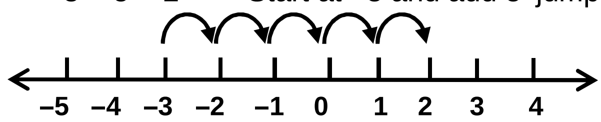
Grade 7, 8 and 9

The integers are … –5; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5 …

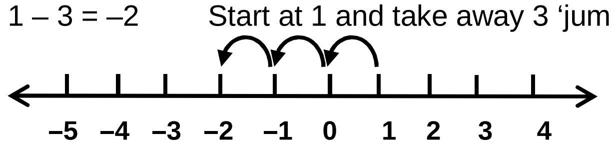
Adding and subtracting integers

We can think about adding or subtracting numbers using a number line.

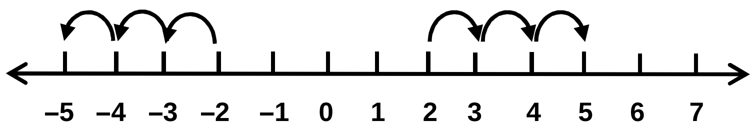
1. –3 + 5 = 2 Start at –3 and add 5 'jumps' to the right.



2. 1 – 3 = –2 Start at 1 and take away 3 'jumps' to the left.



3. Notice that 1 – 3 = –(3 – 1) = –2.
   This helps with bigger calculations e.g. 265 – 378 = –(378 – 265) = –113
4. We can also observe that –2 – 3 = –(2 + 3).
   We can see this easily on the number line.



This helps with bigger calculations e.g. -345-123 = –(345+123) = –468

5. Adding a negative number has the same effect as subtracting a positive number so 20 + (–5) = 20 – 5 = 15
6. Subtracting a negative number has the same effect as adding a positive number so –4 –(–10)= –4 + 10
7. We know that addition is commutative. This means that the order of adding two numbers does not matter e.g. 5 + 7 = 7 + 5.
   So –73 + 95 = 95 + (–73) = 95 – 73 = 22. This makes the calculation easier.

Multiplying and dividing integers

Grade 8 and 9

					Examples
+ positive	× or ÷	+ positive	=	+ positive	3 × 2 = 6 This is 3 jumps of 2 on the number line
+ positive	× or ÷	− negative	=	− negative	3 × (–2) = –6 This is 3 jumps of –2 on the number line
− negative	× or ÷	+ positive	=	− negative	(–2) × 3 = –6 (–2) × 3 must be the same as 3 × (–2)
− negative	× or ÷	− negative	=	+ positive	*The reason for this can be shown mathematically. For now we will just accept it!*

Questions

Unit 1.9: Integers

Grade 7, 8 and 9:

Calculate:

1a)	8 – 10	b)	5 – 12	c)	– 8 – 3	d)	23 – 46	e)	36 – 58
2a)	– 5 – 2	b)	– 7 + 3	c)	– 8 + 10	d)	– 12 – 15	e)	– 32 + 14
3a)	6 – (–3)	b)	8 + (–12)	c)	12 – (–2)	d)	26 + (–14)	e)	28 + (–32)
4a)	9 + (–3)	b)	12 – (–4)	c)	39 + (–4)	d)	48 – (–3)	e)	–12 – (–15)
5a)	240 – (–45)	b)	–36 – 42	c)	–124 + 235	d)	–14 + (–27)	e)	143 – 189
6a)	3 – 5 + 7	b)	–5 – 2 + 7	c)	–23 + 27 – (–4)	d)	2 + (–10) – 24		

7. You may only use the following numbers to answer the questions:
   –7; –5; –2; 2; 5; 7.
   a) Use two of the numbers to make the largest possible result. ☐ + ☐ =
   b) Use two of the numbers to make the smallest possible result. ☐ + ☐ =
   c) Use two of the numbers to make the largest possible result. ☐ – ☐ =
   d) Use two of the numbers to make the smallest possible result. ☐ – ☐ =

8. The temperature in Johannesburg was 7° C at 6 pm in the evening. By 6 am the next morning, it had dropped by 9° C. By noon it had risen by 18° C. How many degrees warmer was it at noon than at 6 pm the day before?

9. Clifford claims that adding two numbers will *always* give him a bigger answer than if he subtracts two numbers. Is he correct? Explain why you say so.

Grade 8 and 9:

Calculate:

10a)	6 × (–3)	b)	–7 × 5	c)	(–5) × (–8)	d)	7 × (–2)	e)	(–8) × (–6)
11a)	18 ÷ (–3)	b)	–24 ÷ 6	c)	(–49) ÷ (–7)	d)	28 ÷ 7	e)	–32 ÷ (–4)

12a)	8 + 3(5 – 7)	b)	24 ÷ (5–8) + 3 × (–2)	c)	(23– (–4)) ÷ 9 – 6
13a)	$\frac{-12}{9} \times \frac{6}{4}$	b)	$\frac{-15}{-5} + 3(7-9)$	c)	$\frac{9+6}{-6} - \frac{1}{2}$
14a)	$(-3)^2$	b)	-3^2	c)	$-3^2 - (-3)^2$

15. Put a number in the box to make the following statements true:

a)	☐ + (–3) = 12	b)	15 + ☐ = 6	c)	15 – ☐ = 6
d)	☐– (–7) = –1	e)	☐– (–5) = 9	f)	235 + ☐ = 189
g)	☐– (–72) = –64	h)	145 – ☐ = 136	i)	☐ + (–13) = 22
j)	☐× (–5) = 15	k)	2 × ☐ = 10	l)	7 × ☐ = 21
m)	☐× 4 = –12	n)	–120 ÷ ☐ = 20	p)	☐× (–12) = 84

16. Given $p = -2$, $q = 7$ and $r = -1$.
    Substitute these values into the expressions and simplify without using a calculator.

a)	$2pq$	b)	$(q-p)$	c)	$(p+q)-p$
d)	$(p-q)-q$	e)	$10r^2 \div p$	f)	$q^2 - pq$
g)	$\frac{1}{2}pq - r$	h)	$3p + 2q$	i)	$(pqr - p^2q^3r^4)^0$

17. Two numbers have a sum of –6 and a product of –16. What are the two numbers?
18. Two numbers have a sum of –17 and a product of 72. What are the two numbers?
19. Two numbers have a quotient of 3 and a product of 12. Give all possible values for the two numbers.

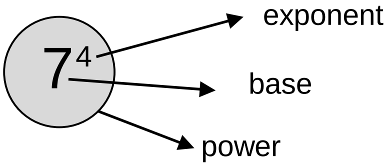
Unit 1.10: Exponents

Grade 7, 8 and 9

Exponential notation

We use exponents to represent repeated multiplication of a number.
The exponent of a number shows how many times to multiply the base by itself.

$7^4 = 7 \times 7 \times 7 \times 7$



Example:

$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

Squares and cubes

A perfect square is a number that has *two identical* factors e.g. $25 = 5 \times 5 = 5^2$
A perfect cube is a number that has *three identical* factors e.g. $64 = 4 \times 4 \times 4 = 4^3$

Square roots and cube roots

The square root of a number is a single factor that, when multiplied by itself, gives the number e.g. $\sqrt{36} = 6$ because 6 × 6 = 36
The cube root of a number is a single factor that, when multiplied by itself 3 times, gives the number e.g. $\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 5^3 = 125$

Useful squares, cubes, square roots and cube roots
These are used often, so you should know them. Then you won't need to calculate them each time you use them.

Learn these!

$1^2 = 1$	$\sqrt{1} = 1$	$7^2 = 49$	$\sqrt{49} = 7$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$8^2 = 64$	$\sqrt{64} = 8$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$9^2 = 81$	$\sqrt{81} = 9$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$10^2 = 100$	$\sqrt{100} = 10$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$11^2 = 121$	$\sqrt{121} = 11$	$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$12^2 = 144$	$\sqrt{144} = 12$	$6^3 = 216$	$\sqrt[3]{216} = 6$

Grade 8 and 9

Scientific notation

Scientific notation uses a shorter way of writing very large or very small numbers. Numbers are converted to a decimal number, with a one-digit whole number (1 to 9), multiplied by a power of 10 with a positive or negative exponent

Examples:

1. 4 000 = 4 × 1 000 = 4×10^3
2. 4 060 = $4{,}06 \times 10^3$
3. 0,07 is 7 × 0,01 = 7×10^{-2}
4. 0,076 = $7{,}6 \times 10^{-2}$
5. $1\ 303\ 000\ 000 = 1{,}303 \times 10^9$
6. $3{,}415 \times 10^8 = 341\ 500\ 000$

The whole number is 4. There are 3 places after the 4, so the exponent is 3.

Make 7 the whole number. 7 is 2 places *after* the decimal comma, so the exponent is negative 2.

9 places after the 1, so the exponent is 9

3 is multiplied by 10^8, so multiply by 10 eight times, which is the same as moving the decimal comma to the right by 8 places.

Laws of operations when working with exponents

Grade 7, 8 and 9

When you calculate with numbers that have exponents, you need to use the following order of operations:

Brackets **E**xponents **D**ivision / **M**ultiplication } **A**ddition / **S**ubtraction }

Remember:
Use BEDMAS and if there is only division and multiplication then just work from left to right.
If there is only addition and subtraction in the calculation, then just work from left to right.

Worked example:

$3 + 6^2 \div 4 - \left(\sqrt[3]{27} + 1\right)$
$= 3 + 6^2 \div 4 - (3 + 1)$
$= 3 + 36 \div 4 - 4$
$= 3 + 9 - 4 = 8$

→ Deal with cube root in bracket first
→ Calculate the **E**xponent and 3 + 1 in **b**racket
→ **D**ivide and then **A**dd and **S**ubtract

Laws of exponents

Grade 8 and 9

x is a natural number; m, n and p are integers

$x^m \times x^n = x^{m+n}$ To multiply powers with *same base*, add their exponents.
$\frac{x^m}{x^n} = x^{m-n}$ To divide powers with same base, subtract their exponents.
$(x^m)^n = x^{mn}$ To raise a power to a power, multiply the exponents.
$(x^m y^n)^p = x^{mp} y^{np}$
$x^0 = 1$ Any base raised to the power of 0 is 1.

Worked example: Simplify:

$\frac{x^{23}y^3 \times xy^7}{(x^4y)^5} = \frac{x^{24}y^{10}}{x^{20}y^5}$
$= x^4y^5$

→ To multiply, add exponents of same bases; multiply power in denominator
→ to divide, subtract exponents of same bases

Laws of negative exponents

Grade 9

A negative power in the numerator is the same as the positive power in the denominator:

$x^{-m} = \frac{1}{x^m}$ and $\frac{1}{x^{-m}} = x^m$

Worked examples:

$\frac{a^6b^{12}c^{-4}}{a^9b^5} = \frac{b^7}{a^3c^4}$ $\frac{a^6}{a^9} = \frac{1}{a^3}$ $\frac{b^{12}}{b^5} = b^7$ $c^{-4} = \frac{1}{c^4}$

Solving simple exponential equations

- Write both sides of the equation with the *same base*.
- If the bases are the same, the powers must be equal.

Worked example:

$9^x 3^x = \frac{1}{27}$
$(3^2)^x 3^x = \frac{1}{3^3}$
$3^{2x} 3^x = 3^{-3}$
$3^{3x} = 3^{-3}$
So $3x = -3$ and thus $x = -1$

Questions

Unit 1.10: Exponents

Grade 7, 8 and 9

1. Calculate:
   a) $3 + 6^2 \div 2$ b) $(30 - 3) \div 3^2$
   c) $\sqrt{25 - 9} \times 2$ d) $3 + \sqrt[3]{27} \times 4$
   e) $(4 - 1 + 8 \div 8) \times 5$ f) $(3 - 1)^3 \div 4$

2. Write in expanded form:
   a) 3^7 b) $6^2.7^4$ c) $\left(\frac{1}{2}\right)^5$

3. Write in exponential notation:
   a) $3 \times 3 \times 3 \times 3$ b) $4 \times 6 \times 6 \times 6 \times 4 \times 4 \times 6$
   c) $\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$ d) (0,1) × (0,1) ×(0,1) × (0,1)

4. Two learners find different answers to a problem.
   Say which one is correct and give a reason.

Learner A	Learner B
$3 \times 3^2 + 4$	$3 \times 3^2 + 4$
$= 3 \times 9 + 4$	$= 9^2 + 4$
$= 27 + 4$	$= 81 + 4$
$= 31$	$= 85$

5. Which is the better offer, A or B? Show your calculations. *You may use a calculator.*

   A. R1 000 in cash today.

   B. Start with 2 cents today, which doubles to 4 cents the next day and 8 cents the following day and so on. ie 2c, 4c, 8c, 16c, 32c ... for 30 days. The amount you receive is the amount for day 30, NOT the sum of the amounts from all the days.

Grade 8 and 9

6. Calculate:
   a) $-4 + (3 - 6)^2$ b) $1 - \sqrt{25} \times 3 + 3^2$
   c) $2 \times 2^2 \div (-4)$ d) $(-5)^3 \div \sqrt{25}$

7. Write in expanded form:
   a) $(-4)^6$ b) $(-2)^7.7^2$ c) x^4y^3

8. Write in exponential notation:
   a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$ b) $(-2) \times (-2) \times (-2)$

9. Write in scientific notation:
   a) 1 260 000 b) 2 320 000 000 000
   c) 103,2 d) $216{,}67 \times 10^4$

10. Simplify, writing answers with positive exponents:

a) $2a^2 \times 3a^3$

b) $\frac{16a^{10}}{4a^4}$

c) $(2x^3)^2$

d) $\frac{6a^3}{24am}$

e) $\frac{2(6xy^3)^2}{3y^3}$

11. Simplify, writing answers with positive exponents:

a) $x^{n+2}.x^2.x^{n+1}$

b) $(3x^3y)^2$

c) $2xy^2 \div (4x^2y^3)^2 \times 8x^4y^5$

d) $\frac{(4p^2q)(-2pq^3)}{(-2pq)}$

e) $[2(3x^5y^{-4}) \times x^{-3}y]^0$

f) $\frac{(a^2m^3)^2(am^2)^2}{a^3m^4}$

12. Write in exponential notation:

a) $x.x.x.x.y.y.y.y.y$

b) $a.a.b.a.b.c.c.b.a.a.b.b.c.c$

Grade 9

13. Write in scientific notation:

a) 0,000 000 078 6

b) 0,06

c) $28{,}8 \times 10^{-9}$

d) 30×10^{-5}

14. Simplify, writing answers with positive exponents:

a) $(3x^{-1})^3$

b) $(a^{-2})(-3a^0)$

c) $\left(\frac{4}{5}\right)^{-2}$

d) $(2ab^3)^{-2}(4a^3b^5)$

15. The speed of light is $299\ 800\ \text{km/s}$. Calculate how far light travels in a year.
Use scientific notation to help you in the calculation.

16. The diameter of the nucleus of a cell is 6 micrometres.
One micrometre = 1 millionth of a metre.
What is the diameter of the cell in metres?

17. The diameter of a virus is 100 nanometres (nm). 1 nm = 10^{-9} metres.
a) What is the diameter of the virus in metres?
b) How many of these viruses could fit along a 1 cm line?

18. Solve for x:

a) $2^x = 8$

b) $3^x = \frac{1}{81}$

c) $6^{x+1} = 1$

d) $2^x 8^x = 16$

Unit 2.1: Algebraic Expressions

Understanding algebraic expressions

Look at $2x + 4$ and read the definitions.

$2x + 4$

Coefficient:
The number that is multiplied by the variable.

Constant:
A number which has one value and does not change. It has no variable part.

Variable:
A letter to represent a number we don't know, or a number that can change.

Algebraic expression:
Made of coefficients, variables, operations and constant numbers.

Terms:
Terms of an expression are separated by + or –.
$2x + 4$ has **two** terms, $2x$ and 4.

Important:

- When a variable is multiplied by a number or another variable, you don't need to use × or a dot '**.**' So $2 \times x = 2 \,.\, x = 2x$
- If there is no number shown next to the variable, you know that the **coefficient** is **1**.
  In $2x + y$, 1 is the coefficient of y.
- If the variable is given a value, then the value of the expression can be calculated.
  So if $x = 3$, then $2x + 4 = 2(3) + 4 = 10$

Examples:

1. $2x + 4$ is two terms
2. $2p - 1$ is two terms
3. $\frac{p-3}{2}$ is one term
4. $3(2x + 4)$ is one term, but $6x + 4$ is two terms!
5. $3a \times 2 - 8$ is two terms

6. In the expression $4x + 3$, x is a variable; 4 is the coefficient of x; 3 is the constant

7. In $2 + a$
   a is a variable; 1 is the coefficient of a; 2 is the constant

8. In $-2y - 1$
   y is a variable; -2 is coefficient of y
   -1 is the constant.

Grade 7, 8 and 9

Like terms in algebraic expressions

Look at this algebraic expression:

Like terms are terms with the *same variables* and the *same exponents.*

$2x^2$ and $2xy$ are *not* like terms

Like terms

$\boxed{2x^2} + 2xy - 4xy \boxed{+ x^2}$

Like terms

$\boxed{2x^2} + 2xy - 4xy \boxed{+ x^2}$
$= 3x^2 - 2xy$

When adding or subtracting like terms, the *variables* and *exponents* in the terms *don't change.*
Only add/subtract the coefficients.

Worked examples:

In each expression, mark the like terms in the same way. You can use lines, colours, shapes etc.

1. $\underline{2x} + \underline{\underline{xy}} - 7y - \underline{\underline{4xy}} + 4y + \underline{x}$

2. $a^2b \boxed{+ 2ab^2} + 3a^2b \boxed{- ab^2}$

Mark the like terms and then simplify:

3. $2x \boxed{+ 3} - x \boxed{- 2}$
$= 2x - x + 3 - 2 = x + 1$

Constant terms are like terms.

$\underline{x^2y} + \underline{\underline{2xy^2}} - \underline{\underline{4xy^2}} + \underline{8x^2y} = 9x^2y - 2xy^2$

Grade 8 and 9

Polynomials

- A polynomial is an algebraic expression involving a sum of powers in one or more variables multiplied by coefficients.
- A binomial is a polynomial with exactly 2 terms e.g. $x + 2$ or $x^2 + 3x$
- A trinomial is a polynomial with exactly 3 terms e.g. $x^2 + 2x + 1$

Write the terms in order from the <u>biggest</u> exponent to the <u>smallest</u> exponent of the same variable.

<u>Example</u>: $6x^4 - 10x^2 + 7x - 5$
(exponents are 4, 2, 1 and 0)

Grade 9

Questions

Unit 2.1: Algebraic expressions

Grade 7, 8 and 9

1. Identify the variable, the coefficient of the variable and the constant term in each algebraic expression below.

	variable	coefficient	constant
a) $5x - 8$			
b) $2a + 9$			
c) $6 - 3b$			
d) $2x + 8$			

2. Write an algebraic expression for the following:
If I am x years old now
a) my age in 5 years' time b) my age 5 years ago
c) my father's age if he is twice as old as me
d) my daughter's age if she is half as old as me

3. Change the following descriptions in words into algebraic expressions:
a) The number of hours in d days. b) The number of months in x years.
c) The amount I will pay to use the internet for m minutes, if the internet café charges me R10 to register to use their computers and 50c per minute for internet time.
d) The area of a square that has a perimeter of length p.

4. Calculate the value of the expressions if $x = 2$ and $y = 7$:
a) $2x^2 + y$ b) $2x^3 - y$
c) $xy - 3x$ d) $x - 3xy$
e) $\frac{y}{3} + x$ f) $\frac{x}{3} + y$

Grade 8 and 9

5. Identify (i) the coefficient of x^2 and (ii) the constant in each of these expressions
a) $3x^2 + 2x - 1$ b) $-x^2 + 2x + \frac{1}{2}$ c) $2x + x^2 + 0{,}2$

6. Determine the number of terms in each expression:
a) $2x^3 - x^2 + 4x - 1$ b) $2(p^2 - 1) + m(p^2 - 1)$
c) $(x - 1)(2x + 7)$ d) $\frac{2y-1}{4} + 1$ e) $\frac{x^2-2x+1}{x-1}$

7. Simplify the following expressions by combining like terms
a) $2p + p - p + p^2$ b) $3x^2 + 2x + 5x^2 + 4$ c) $xy^2 + 2xy + y^2 - xy - 3y^2$
d) $3xy + 2yx$ e) $p^3q + pq + p^2q + pq$ f) $a^3 + a^2 + (a \times a)$

8. Calculate the value of the expressions if $a = -1$ and $b = 3$
a) a^b b) $3a^b - 6b$
c) $3ab^2$ d) $(3ab)^2$

9. Complete the pyramid by filling in the missing algebraic expressions.
Each block is the sum of the two blocks below it.

$5a + 28b + 13c$				
$5a + 4b$				
$5a$	$4b$	$2b$	$3c$	c

Unit 2.2: Calculating algebraic expressions

Multiplication:

To multiply, you *don't need like terms.* Any terms can be multiplied together.

STEPS:

Worked example: $3x^2z \times x^2y^3 \times (-4xy^3)$

1. Multiply the coefficients (with their signs). → ✓ $3 \times 1 \times -4 = -12$
2. Multiply the variables with the same base by adding the exponents. → ✓ $x^2 \times x^2 \times x = x^5$ and $y^3 \times y^3 = y^6$
3. If there are powers that don't have the same bases, write them in your final answer leaving out the × sign. → ✓ $-12\,x^4z\,y^6$
4. If your final answer has more than one variable, list them in alphabetical order. → ✓ $-12\,x^4\,y^6\,z$

Worked examples: Simplify the following expressions

1. $2m \times -3m^2$
   $= -6m^3$

2. $x^3y^4 \times xyz^2$
   $= x^4y^5z^2$

Grade 8 & 9

The distributive property:

Multiplication is distributed over addition and subtraction. You may have used this property before to multiply numbers:

$$8 \times 36 = 8(30 + 6) = 8 \times 30 + 8 \times 6 = 240 + 42 = 282$$

Remember BEDMAS.
Multiply before adding.

Worked examples: Simplify the following:

1. $3(2x + 1) = 6x + 3$

2. $-3a(a^2 + 4a - 2) = -3a^3 - 12a^2 + 6a$

Be careful with the + signs and the – signs

Product of two binomials:

Multiply both terms in the first bracket by both terms in the second bracket.

F: first terms
O: outer terms
I: inner terms
L: last terms

Worked examples:
Simplify the following

1. F L
   $(x + 3)(2x - 1)$
   I
   O

   $= 2x^2 - x + 6x - 3$
   $= 2x^2 + 5x - 3$

First multiply, then look for like terms to add. In this example, $-x$ and $+6x$ are like terms which simplify to $+5x$.

2. $(2a - b)(4a - 3b)$
   $= 8a^2 - 6ab - 4ab + 3b^2$
   $= 8a^2 - 10ab + 3b^2$

Grade 9

Squaring a binomial

Look at the expression $(x+3)^2$.

$(x+3)$ is the base and it is squared. So $(x+3)$ is multiplied by itself.

So, $(x+3)^2 = (x+3)(x+3)$

Worked examples: Simplify

1. $(a+1)^2$

$= (a+1)(a+1)$

$= a^2 + a + a + 1$

$= a^2 + 2a + 1$

Use FOIL as before.

2. $(2y-3)^2 = (2y-3)(2y-3)$

$= 4y^2 - 6y - 6y + 9$

$= 4y^2 - 12y + 9$

Use FOIL as before.

Grade 9

The product of the sum and difference of binomials:

When two binomials have the same numbers, but different signs, the product will be the difference of two squares.

Worked examples: Simplify

1. $(m+4)(m-4)$

$= m^2 + 4m - 4m - 16$

$= m^2 - 16$

2. $(3p-4q)(3p+4q)$

$= 9p^2 - 16q^2$

If the brackets have opposite signs and the same variables and constants, the middle terms will always add up to 0. When you see this pattern, you can work quickly by just multiplying **Firsts** and **Lasts** in each bracket.

The final answer is a *difference* (–) *of perfect squares*.

Dividing algebraic expressions

Dividing is the same as simplifying algebraic fractions.

We can split fractions that have only one denominator.
For example, $\frac{5+1}{8}$ can be separated into $\frac{5}{8} + \frac{1}{8}$
This is useful for dividing algebraic expressions.

STEPS:

- ✓ If there is more than one term in the *numerator*, split the fraction as shown. →
- ✓ Simplify the coefficients of each fraction if possible. →
- ✓ Where the bases are the same, simplify the powers by subtracting the exponents. →

Worked example:

Simplify: $\frac{3p^3+2pq^2}{6pq}$

$\frac{3p^3+2pq^2}{6pq}$

$= \frac{3p^3}{6pq} + \frac{2pq^2}{6pq}$

$= \frac{1p^3}{2pq} + \frac{1pq^2}{3pq}$

$= \frac{p^2}{2q} + \frac{q}{3}$

Grade 8 & 9

Worked examples: Simplify

1. $\frac{3m^4n^5}{9mn^2}$
$= \frac{m^3n^3}{3}$

2. $\frac{2x^3+3xy^2}{6}$
$= \frac{2x^3}{6} + \frac{3xy^2}{6}$
$= \frac{x^3}{3} + \frac{xy^2}{2}$

3. $\frac{4a^2b^7c^3+ab^4c-5ab^6c^2}{10ab^3}$
$= \frac{4a^2b^7c^3}{10ab^3} + \frac{ab^4c}{10ab^3} - \frac{5ab^6c^2}{10ab^3}$
$= \frac{2ab^4c^3}{5} + \frac{bc}{10} - \frac{b^3c^2}{2}$

You can only simplify powers that have the same bases (a's with a's; b's with b's, c's with c's).

Square roots of algebraic expressions:

✓ The terms under the square root have the same bases, so we can simplify by adding like terms. We *must* add the like terms first and find the square root of a single term. You *cannot* square root each term first.

✓ To find the square root of a variable power, divide the exponent by 2.

Worked example:

$\sqrt{9a^{16} + 16a^{16}}$
$= \sqrt{25a^{16}}$
$= 5a^8$
(because $5 \times 5 = 25$ and $a^8 \times a^8 = a^{16}$)

Worked examples: Simplify

1. $\sqrt{25x^2} = 5x$
2. $\sqrt{4y^4 + 9y^6}$

You can't simplify further because y^4 and y^6 are *not* like terms and you can't find the root of each term separately.

Cube roots of algebraic expressions:

✓ There must be a single term under the cube root sign, so first add like terms.

✓ To find the cube root of a variable power, divide the exponent by 3.

Worked examples:
Simplify

1. $\sqrt[3]{-64m^9} = -4m^3$

2. $\sqrt[3]{21x^6y^3 + 6x^6y^3}$
$= \sqrt[3]{27x^6y^3} = 3x^2y$

Grade 8 & 9

Questions

Unit 2.2: Multiplying algebraic expressions

Grade 8 and 9

1. Simplify the following expressions:

a) $-2x^2 + 3x - 4 - 10x + x^2$

b) $m^2n + 2mn + 4mn - 3m^2n + mn^2$

c) $-2b + 5 + b^2 + b + 4 + 2b^6 + 2b - 8$

d) $-4pq + 2p - 3q - 2pq + 5q$

e) $(2a^2bc^3)(-a^2b^5)$

f) $2y(y^2 + 4y)$

g) $(m^2)[2(m + 1)]$

h) $-mn(m^2 + 3m + 4n)$

i) $4(3x + 2) + 5(x + 1)$

j) $xy(2x + 3y) + 5(x^2y - xy^2)$

k) $10x^2 - x(5x + 2) + 3(x - 1)$

l) $-(a - 3) - (3 - a)$

m) $-2x(x + y) + 3x(x - y)$

n) $\frac{1}{2}(6x - 4) + \frac{9x+12}{3}$

o) $\frac{4x^2y^3}{12xy}$

p) $\frac{5a^3b^2c - 3a^2b}{15a^2b}$

q) $\frac{3x^2 - 6y + 9}{3}$

r) $\sqrt{49p^{16}}$

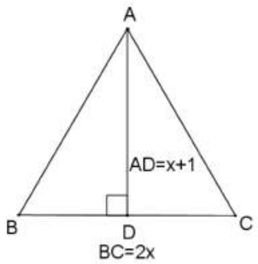
s) $\sqrt{x^6y^2}$

t) $\sqrt{25a^4b^8 - 9a^4b^8}$

u) $\sqrt[3]{-125x^3}$

v) $\frac{8a^2b + 14ab^2 + 3a^2b^3}{2ab}$

2. Calculate the area of the triangle in terms of x.
Area $\Delta = \frac{1}{2}.b.h$



Grade 9

3. Simplify the following expressions:

a) $(a + 4)(2a + 1)$

b) $(x - 3y)(2x + y)$

c) $(m + n)(3m + 3)$

d) $(y + 3)^2$

e) $(b - 2c)^2$

f) $(x - 3)(x + 3)$

g) $(2a - 3b)(2a + 3b)$

h) $(\frac{1}{2}x + 3)(x - 4)$

i) $2(x - 5)(x + 5)$

j) $-(x + 5)(x - 3)$

k) $(7 - 3x)^2$

l) $(x + 3)(x - 2) - (x + 1)(x + 2)$

m) $(3x + 4)(x - 1) - (x + 1)^2$

n) $-0{,}1(x + 30)(10x + 20)$

Find the product:

4. $(x - a)(x - b)(x - c)(x - d) \ldots (x - x)(x - y)(x - z)$

Unit 2.3: Factorising algebraic expressions

When you factorise, you are given a product and you need to find the factors that were multiplied together to get that product. For example, the number 45 has the factors 9 and 5. So you need to find the values that were multiplied together to get the given value.
We look at three ways to factorise an algebraic expression:

Taking out the highest common factor (HCF):

STEPS

Worked example: $2x^3 - 6x + 2x$

- ✓ Add any like terms. → $= 2x^3 - 4x$
- ✓ Look for the highest common factor to all the terms in the expression. → HCF is $2x$

  The lowest power of x that is common to x and x^3 is x
- ✓ Write the HCF in front of a bracket. → $= 2x(\quad)$
- ✓ The terms in the bracket are the factors that the HCF must be multiplied by to get back to the original expression. → $= 2x(x^2 - 2)$

  because $2x \times x^2 = 2x^3$ and $2x \times -2 = -4x$

Worked examples: Factorise fully

1. $3y + 12$
   $= 3(y + 4)$

   The second term has no y so just 3 is the HCF

2. $4a^2b^3 - 2ab^5 + a^2b^4c$
   $= ab^3(4a - 2b^2 + abc)$

   The variable c is *not* common as it is in one term only. There is no common coefficient.

3. $\underbrace{2(\boldsymbol{p-3})}_{\text{Term 1}} - \underbrace{3q(\boldsymbol{p-3})}_{\text{Term 2}}$

   $= \underbrace{(\boldsymbol{p-3})}_{\text{common factor}}\ (2 - 3q)$

   There are 2 terms and $p - 3$ is common to both terms. Take it out as the common factor.

The difference of two squares:

Example: $2x^2 - 50$

- ✓ Check for an HCF. If there is one, take it out first. → $= 2(x^2 - 25)$

  $\sqrt{25} = \pm 5$
- ✓ If the expression is the difference of two perfect squares, then factorise into a sum and difference of the square roots. → $2\underbrace{(x+5)}_{sum}\underbrace{(x-5)}_{difference}$
- ✓ Remember to check if a factor can be factorised further. → No more factors

Worked examples: Factorise fully

1. $a^4 - 16$
   $= (a^2 + 4)(a^2 - 4)$
   $= (a^2 + 4)(a + 2)(a - 2)$

   $a^2 + 4$ is the *sum* of two squares and *cannot* be factorised. $a^2 - 4$ is another difference of squares and must be factorised again.

3. $2m^2 - 8n^4$
   $= 2(m^2 - 4n^4)$
   $= 2(m + 2n^2)(m - 2n^2)$

   First take out the common factor of 2. The bracket is a difference of squares and must be factorised again.

Grade 9

Quadratic Trinomials:

STEPS

- ✓ Take out the HCF if possible.
- ✓ If the expression (or the factor in the bracket) is in the form x^2+bx+c, it is a *quadratic trinomial* and may factorise into two brackets.
- ✓ List the pairs of factors of the last term c
- ✓ Check if any pair adds up to b (the coefficient of the middle term).
- ✓ These factors will be the constant terms in the brackets.

Example:

$3x^2+15x+18$

→ $=3(x^2+5x+6)$

→ Factors of 6:
1 and 6, 2 and 3.

→ 2 + 3 = 5

→ $=3(x+\mathbf{2})(x+\mathbf{3})$

Examples: Factorise fully

1. a^2-6a+8
$=(a-4)(a-2)$

   -4 × -2 = 8 (last term)
   -4 - 2 = -6 (middle term)

2. $m^2+4m-12$
$=(m+6)(m-2)$

   6 × -2 = -12
   6 - 2 = 4

3. $y^2-5y-14$
$=(y+2)(y-7)$

   2 × -7 = -14
   2 - 7 = -5

4. $2b^2-10b-12$
$=2(b^2-5b-6)$
$=2(b-6)(b+1)$

   -6 × 1 = -6
   -6 + 1 = -5

Must I use +ve or –ve?

Sign of middle term	Sign of constant	Signs in brackets
x^2 +	$bx+c$	(+)(+)
x^2 –	$bx+c$	(–)(–)
x^2 +	$bx-c$	(+big)(–small)
x^2 –	$bx-c$	(–big)(+small)

Check your factors carefully:

- ✓ When you × the factors, they must give you the *last* term of the trinomial.
- ✓ When you + the factors, they must give you the coefficient of the

Using factorisation to simplify algebraic expressions:

- ✓ Before you can simplify an algebraic fraction, it must have only *one term* in both the numerator and denominator.
- ✓ Factorise fully at the top and the bottom of the fraction to get one term in each.
- ✓ Simplify by dividing factors that are the same.

Example:

$\frac{2p^2+4p}{3p+6}$ (two terms at top and at bottom so need to factorise)

$=\frac{2p\cancel{(p+2)}}{3\cancel{(p+2)}}$

$=\frac{2p}{3}$

Examples: Simplify

1. $\frac{x^2-25}{x^2+8x+15}\div\frac{2x-10}{x^2-9}$
$=\frac{\cancel{(x-5)}\cancel{(x+5)}}{\cancel{(x+5)}\cancel{(x+3)}}\times\frac{\cancel{(x+3)}(x-3)}{2\cancel{(x-5)}}$
$=\frac{x-3}{2}$

   Change the divide to multiply and invert (tip) the second fraction.

   Factorise and simplify. You can cancel common factors from any numerator with any denominator

2. $\sqrt{x^2+10x+25}$
$=\sqrt{(x+5)^2}$
$=x+5$

   Factorise the trinomial
   $(x+5)^2$ is a perfect square

Grade 9

Questions

Unit 2.3: Factorising algebraic expressions

Grade 9

Factorise fully:

1. $3x - 9$
2. $3a^4 - 4a$
3. $11a^2x - 22ax^2 + 55ax$
4. $m(p - 3q) - 4(p - 3q)$
5. $ab - ac$
6. $2a^2 - 6ab$
7. $x^3 - x^2$
8. $3p^2 + 6p^3 + 9p^6$
9. $4x(2x + 5) - (2x + 5)$
10. $1 - 16a^2$
11. $x(3x + 2) + 5(3x + 2)$
12. $2(x - y) + 3(y - x)$
13. $7(b - 2a) + 3(2a - b)$
14. $x^2 + 5x - 24$
15. $x^2 + 6x + 8$
16. $5x^2 - 30x + 45$
17. $\frac{1}{2}x^2 + 3x + 4$
18. $2y^2 + 12y + 18$
19. $2x^2 + 8x - 42$
20. $y^4 + 2y^2 + 1$
21. $25k^2 - 9m^4$
22. $x^2 + 9x + 14$
23. $x^2 - 14x + 48$
24. $x^2 - x - 2$
25. $x^2 + 4x - 45$
26. $2x^2 - 18y^2$
27. $4x^3 + 28x^2 - 32x$
28. $5x^2 - 5y^2$
29. $32x^2y - 50y^3$
30. $bx^2 - by^2$
31. $49y^6 - \frac{1}{9}x^2$
32. What are the dimensions of a square with an area of $x^2 + 2x + 1$?

Simplify the following expressions:

33. $\frac{2m^2+4m}{m^2-4}$
34. $\frac{a^2+a}{a+1}$
35. $\frac{x^2-25}{x^2-2x-15}$
36. $\frac{x^2+2x+1}{x^2-1}$
37. $\frac{a^2-a}{a^2-2a+1} \times \frac{3a-3}{a}$
38. $\frac{2x^2+6x+4}{x^2+5x+6}$
39. $\frac{x^4-81x^2}{2x} \div \frac{x^2-9x}{6}$
40. $\frac{x^4-1}{x^2+1}$

Unit 2.4: Algebraic equations

Algebraic equations

$3x + 9 = 27$

same value

Worked examples:

1. John is 16. He is 4 years older than Thando. Let Thando's age be x.

   So John's age is x + 4 years.
   Then we can find Thando's age with an equation: $x + 4 = 16$

2. The sum of two consecutive numbers is 77.

   Let the smaller number be y.
   The next consecutive number is $y + 1$.
   We can use the equation: $y + (y + 1) = 77$
   to find y.

   > **Consecutive** numbers are numbers in counting order that follow on eg 5, 6 and 7.

3. Six buses were used to take 331 learners on a trip. 7 learners had to go in a taxi because the buses were full. There are x seats on each bus.

   6 buses with x seats each is 6x seats. There are 7 more learners.
   So we can write the equation $6x + 7 = 331$ to find x, the number of seats on each bus.

Ways to solve equations

Worked examples:
Solve for the variable.

→ *By inspection (mental mathematics)*

1. $x + 4 = 10$ can be read as *"What number, when you add 4 to it, makes 10?"*
   We can see the number is 6.

2. $4 = 8 - 2a$ can be read as *"What number, when you double it and subtract it from 8, makes 4?"*
   We know 8 – 4 = 4 (or 4 = 8 – 4), so $2a = 4$ and $a = 2$

Grade 7, 8 and 9

Solving equations using inverse operations:

If you add, subtract, multiply or divide on one side of the equation, you must do the same to other side to keep the balance. Whatever you do to one side of the equation, you must do to the other side.

How can you answer this question?

Solve for x if $3x + 9 = 27$

To find the value of x in this equation, try to get x on its own on one side of the equation.

Subtract 9 on both sides first:

$3x + 9 \mathbf{- 9} = 27 \mathbf{- 9}$

$3x = 18$

+ and – are inverse operations

Divide both sides by 3:

$\frac{3x}{\mathbf{3}} = \frac{18}{\mathbf{3}}$

$x = 6$

$3x$ means $3 \times x$, so we need to divide (the inverse operation of multiply)

So $x = 6$ is the answer to the equation.

- Only one = on each line.
- Keep the = signs underneath each other so you can see the left and right sides easily.

Worked examples: Solve for the variable.

1. $2(p-3) = 3p + p$

$\therefore 2p - 6 - \mathbf{2p} = 4p - \mathbf{2p}$

$\therefore -6 = 2p$

$\therefore \frac{-6}{\mathbf{2}} = \frac{2p}{\mathbf{2}}$

$\therefore -3 = p$

- First simplify both sides.
- Choose to subtract $2p$ (or you can add 6). *Do this on both sides!*
- $2p$ means $2 \times p$, so the inverse is to divide by 2.

2. $\frac{2x}{5} = 3$

$\frac{2x}{5} \times \mathbf{5} = 3 \times \mathbf{5}$

$\therefore 2x = 15$

$\therefore \frac{2x}{\mathbf{2}} = \frac{15}{\mathbf{2}}$

$\therefore x = \frac{15}{2}$

The inverse of dividing $2x$ by 5 is to multiply by 5.
Do the same on the RHS.

Solving equations with fractions

To solve equations with fractions, get rid of the fractions by multiplying both sides of the equation by the lowest common denominator of all the fractions in the equation.

Worked example: Solve $\frac{2x+1}{6} + \frac{x-5}{3} = -1$

TIP: Use brackets in the numerator to show that the whole expression is divided by the denominator.

$\frac{(2x+1)}{6} + \frac{(x-5)}{3} = -1$

$6 \times \left(\frac{(2x+1)}{6} + \frac{(x-5)}{3}\right) = -1 \times 6$

The lowest common denominator of 6 and 3 is 6. Multiply every term on both sides of the equation by 6.

$2x + 1 + 2(x-5) = -6$

$2x + 1 + 2x - 10 = -6$

$4x - 9 = -6$ so $4x = 3$ so $x = \frac{3}{4}$

Grade 8 & 9

Solving quadratic equations using factorisation:

Grade 9

A quadratic equation has x^2 (or p^2, m^2 etc) as its highest power of x, p, m etc.
These are all quadratic equations:

$x^2 - 7x + 10 = 0$ $3p^2 - 7p = 12$

$3m(m - 9) + 2 = 5m$ (When you multiply out, there is a $3m^2$)

Worked example: Solve the quadratic equation.

This means find the values of x that make the equation true.

$x^2 - 7x + 10 = 0$

Factorise to get ( )( ) = 0

$(x - 5)(x - 2) = 0$

If (A) × (B) = 0, then A = 0 or B = 0
Each factor equals 0

Then $x - 5 = 0$ or $x - 2 = 0$

$x = 5$ $x = 2$

Quadratic equations can have 2 possible solutions for the variable.

Worked examples: Solve for the variable

1. $x^2 - 9 = 0$

$\therefore (x+3)(x-3) = 0$

$\therefore x + 3 = 0$ OR $x - 3 = 0$

$\therefore x + 3 - \mathbf{3} = 0 - \mathbf{3}$ OR $\therefore x - 3 + \mathbf{3} = 0 + \mathbf{3}$

$\therefore x = -3$ OR $\therefore x = 3$

2. $x^2 - 2x = 35$

$x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

$\therefore x - 7 = 0$ OR $x + 5 = 0$

$x = 7$ OR $x = -5$

Check your solutions:

Grade 7, 8 and 9

You can check that your answers (solutions) are correct. Substitute your answer into the left side and the right side of the equation given in the question. If the two sides are equal, then your solution is correct.

Examples: Check if the solution given is correct

1. $a + 5 = 4a - 10$. The solution given is $a = 5$

Check: $\text{LHS} = a + 5$ $\text{RHS} = 4a - 10$

$= 5 + 5$ $= 4(5) - 10$

$= 10$ $= 10$

$\therefore a = 5$ is the correct solution

Substitute a = 5 in LHS and in RHS.
Do you get the same answer on both sides?

2. $\frac{3y}{2} = 2y + 7$. The solution given is $y = 1$

Check: $\text{LHS} = \frac{3y}{2}$ $\text{RHS} = 2y + 7$

$= \frac{3(1)}{2}$ $= 2(1) + 7$

$= \frac{3}{2}$ $= 9$ $\therefore y = 1$ is NOT a solution

If answers are different, calculate the original equation again.

Questions

Unit 2.4: Algebraic equations

Grade 7, 8 and 9: Routine questions

Write an equation to represent the situation and then solve the equation:

1. The sum of two consecutive even numbers is 82. The smaller number is x.
2. A pencil costs $\mathrm{R}y$ (y rand). You buy 4 pencils and a book costing R5. The total cost is R25.
3. The consumer studies class bakes x apple pies and the tuckshop bakes 4 apple pies for a cake sale. All the pies are cut into 5 pieces. There are 60 pieces of pie altogether.
4. Jabulani thinks of a number x. Sihle multiplies this number by 6 and John multiplies it by 3 and adds 9. Sihle and John get the same answer. What number did Jabulani think of?
5. Solve these equations by inspection:
   a) $y+3=5$
   b) $m-4=0$
   c) $2z+3=11$
   d) $3a=a+2$
   e) $2r=10$
   f) $x+\frac{1}{2}=4$
   g) $\frac{p}{3}=8$
   h) $\frac{1}{2}x=3$
   i) $0{,}1y=3$
   j) $\frac{1}{3}x-1=1$
   k) $-2x=-6$
   l) $-2x=10$
6. Is $x=3$ a solution of the equation $x^2+2x=15$?
7. Is $x=4$ a solution of the equation $x^2+\sqrt{x}=20$?
8. Thando thinks of a number. Ayanda multiplies this number by 8 and Zelethu multiplies it by 6 and adds 14. Ayanda and Zelethu get the same answer. What number did Thando think of?

Grade 8 and 9: Routine questions

9. Solve these equations for the variable:
   a) $\frac{3x}{4}=2$
   b) $2x-1=3x$
   c) $5y-3=2y+9$
   d) $2(a-2)+a-2=6$
   e) $\frac{3b}{2}=b-1$
   f) $\frac{y}{3}-\frac{y}{2}=4$
   g) $\frac{z-1}{3}=4$
   h) $\frac{3}{x}=6$
   i) $3(x-5)-2(x-1)=7$
   j) $5(x+1)-(x+2)=-(x-3)$
   k) $1-x+(2x-1)=4$
   l) $3(x+1)-2(x+3)=5$
   m) $6x+2=3x-9$
   n) $5(x+2)-2(x-9)=3(x+2)+x$
   o) $\frac{x}{4}+\frac{x}{3}=7$

10. I buy a cup of coffee for R8 and some biscuits that cost R2 each. I pay R20 in total. How many biscuits do I buy?
11. If I subtract 1 from a number, I get the same answer as I do when I add three to that number and then double the result. What is the number?
12. The numbers $x+1$ and 3 have a product of $2x-3$. Find the value of x.
13. The perimeter of a rectangular field is 100 m. The length of the field is 10 m longer than the breadth of the field. How long is the field?
14. I run on Mondays, Wednesdays and Saturday. On Wednesday, I ran 4 km further than I did on Monday. On Saturday, I ran double the distance I did on Monday. In total I ran 40 km in the week. How far did I run on Monday?
15. The numbers $x+1$ and 3 have a product of $2x-3$. Find the value of x.

Grade 9: Routine questions

16. Solve these equations for the variable:

a) $\frac{a}{3}-\frac{a}{5}=\frac{1}{9}$
b) $\frac{x}{5}-8=\frac{x}{3}$
c) $\frac{y}{10}+\frac{y-6}{3}=9$
d) $\frac{b}{4}+\frac{b-3}{9}=9$
e) $\frac{3a}{7}-7=\frac{a-2}{8}$
f) $x^2=64$
g) $x(x-3)=0$
h) $a^2-16=0$
i) $b^2-3b-40=0$
j) $2y^2+6y=0$
k) $x^2+7x+10=0$
l) $k^2=-4k-4$
m) $3k^2+72=33k$
n) $a^2+18a+97=-2a-3$

17. Solve these equations for the variable:

a) $(3x+5)(3x+7)=24x+31$
b) $\frac{p+2}{p-6}=\frac{p+9}{p+5}$
c) $\frac{x-5}{x-9}=\frac{x-10}{x-5}$
d) $x^2-2x=3$
e) $5x^2-10x=-5$
f) $x^2=x+6$
g) $\frac{x-2}{5}-\frac{2x+1}{3}=-2$
h) $\frac{5(x+1)}{3}-x=\frac{x-1}{2}$
i) $\frac{2(x-1)}{4}-3x=x-1$
j) $\frac{x^2+2x+1}{3x+3}+\frac{2x+1}{5}=1$

18. The product of a number and five more than that number is 36. What is the number?

Unit 2.5: Patterns and Algebra

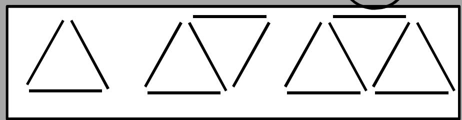
→ **Number pattern:** A list of numbers that are ordered according to a rule is called a number pattern or a number sequence.
→ **Geometric pattern:** A pattern made of shapes according to a rule.

Worked example 1:
Here is a geometric pattern of triangles made from sticks. We can show the same pattern of sticks in words, with numbers, in tables or with a formula that gives the rule of the pattern.

Each step of the pattern is called a **term** (T). The number of the term shows its position in the sequence.

Geometric Pattern

T_1 T_2 T_3



Describe the number of sticks in words:
Start with 3 and add 2 each time to get the next term.

Number pattern
The number pattern of the **number of sticks** used in each step is **3; 5; 7**.
The next step of the pattern would be **9**.
Note:
The number pattern of **triangles** is **1; 2; 3**.

The next term, T_4, will be

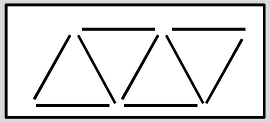


Table to represent the pattern:

position	1	2	3	4
term (no of sticks)	3	5	7	9

The constant difference between the output values is 2.

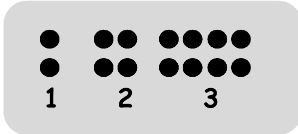
Rule:
Look at the table. What rule will work to get 1 → 3; 2 → 5; 3 → 7; 4 → 9.
Test different numbers.

Double the number and add 1
So the rule is $T_n = 2n + 1$

Note: If there is a constant difference in output values, then that difference will be the coefficient of n.

Grade 7, 8 & 9

Worked example 2:



First try to answer the questions. Then check your answers.
Given the geometric pattern of dots:

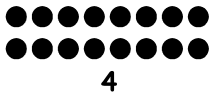
1. Draw the next shape in the pattern.
2. What number can you multiply by each time to get the next term?
3. Explain the pattern in one sentence.
4. Write a number pattern for the number of dots in the first 5 steps of the pattern.
5. Complete the table of values:

Position (term number)	1	2	3	4	6	
Number of dots	2	4				512

6. Using T_n, write down a rule for finding any term of this pattern.

Solutions:

1. Double the number of dots each time:

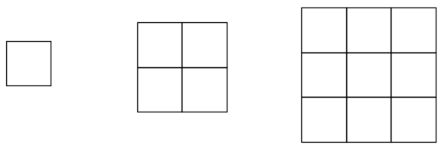


2. Multiply each term by 2 eg $2 \times 2 = 4$; $4 \times 2 = 8$ and $8 \times 2 = 16$
3. Start with 2 and multiply each term by 2 to get the next term. OR
   Start with 2^1 and raise each term of 2 to the power of its position number in the pattern eg $T_2 = 2^2$; $T_3 = 2^3$; $T_4 = 2^4$
4. 2; 4; 8; 16; 32.
5.

Position (term number)	1	2	3	4	6	**9**
Number of dots	2	4	**8**	**16**	**64**	512

6. $T_n = 2^n$

Worked example 3:



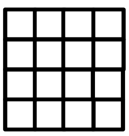
Given the geometric pattern:

1. Draw the next shape in the pattern.
2. Does the pattern use a constant difference or a constant ratio?
3. Explain the pattern in words.
4. Write a number pattern for the number of squares in the first 5 steps of the pattern.
5. Complete the table of values:

Position (term number)	1	2	3	4	7	
Number of squares	1	4				81

6. Using T_n, write a rule for finding any term of this pattern.

Solutions:

1. 
2. There isn't a constant value that you can add each time or a constant multiple between the terms, so we must look for a different pattern.
3. Square the position number to get the term.
4. 1; 4; 9; 16; 25.
5.

Position (term number)	1	2	3	4	7	9
Number of squares	1	4	9	16	49	81

6. $T_n = n^2$

Grade 7, 8 & 9

Questions

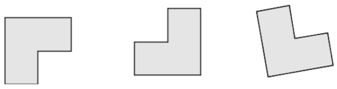
Unit 2.5: Patterns

Grade 7, 8 and 9:

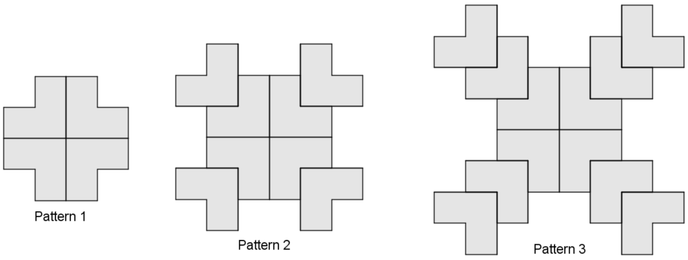
1. For each of the following patterns:
   i) Extend the pattern by 3 terms
   ii) Describe the pattern in words
   iii) Find the general rule for patterns a) and b) only Grade 8 and 9
   You may use a calculator.

a) 1; 4; 7; 10; …	$5\frac{2}{3}$; $5\frac{1}{3}$; 5; $4\frac{2}{3}$; …
b) 20; 15; 10; 5; …	0,4; 0,8; 1,2; 1,6; …
c) 3; 6; 12; 24; …	0,4; 0,8; 1,6; 3,2; …
d) 2; 8; 18; 32; …	243; 81; 27; 9; …
e) 2 187; 729; 243; 81; …	2; –4; 8; –16; …

2. Owen has some tiles like these:



He uses them to make this pattern:



a) How many tiles does Owen add every time he makes a new pattern?
b) How many tiles will he need to make pattern 6?
c) Owen uses 40 tiles to make a pattern. What pattern number is this?
d) Find the general rule for this pattern where n is the pattern number and T_n is the number of tiles. Grade 8 and 9

3. a) Complete the table:

Number of cars (n)	1	2	3	4	5
Number of wheels (T_n)	4	8			

b) Describe the pattern in words.
c) How many wheels do 100 cars have?
d) How many wheels do n cars have? Grade 8 and 9

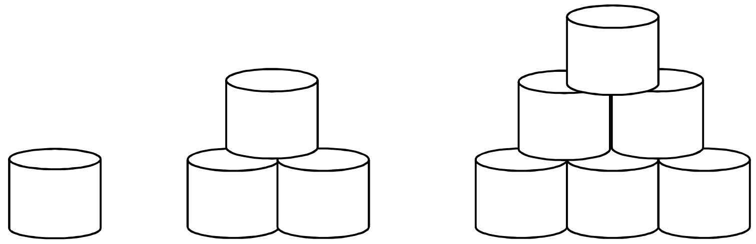
4. Each row of the number pattern below is a *palindrome* (a sequence that is the same if you read it forwards or backwards).

1
121
12 321
1 234 321

a) Add 4 more rows to the pattern.
b) Find the sum of every row.
c) What kind of number is the sum of every row?
d) Write down a rule you can use to find the sum of every row.
e) Use this rule to find the sum of the 40th row.

Grade 7, 8 and 9

5. Samuel works at the supermarket. He must build up a display of tins so that each tin rests on two tins below it:



He continues to build up the display in this way until he has packed all the tins.

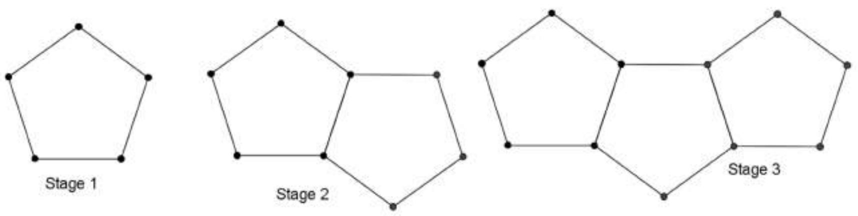
a) Complete the table:

Tins at base	1	2	3	4	5	6
Tins in step of display	1	1 + 2 = 3				

b) Describe the pattern in words.

Grade 8 and 9

6. This pattern of pentagons is made with matches:



a) Write the number pattern for the number of matches in each pentagon up to 6 terms.

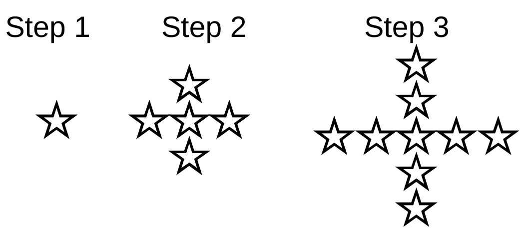
b) Determine the general rule that describes this pattern.

c) Use your general rule to find:

i) The number of matches used in 12 pentagons.

ii) The number of pentagons if 93 matches are used.

7. I draw a pattern of stars. The first three steps of the pattern are shown:



Which step of the pattern will have 241 stars?

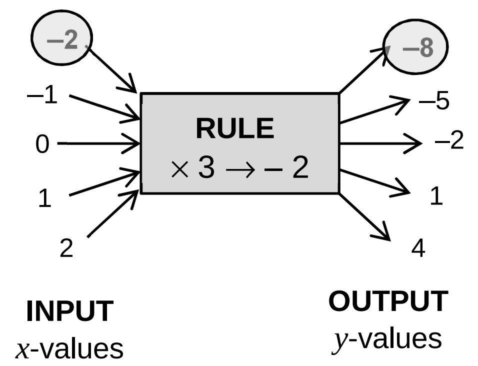
Unit 2.6: Functions

Functions and relations (just like number patterns) use a rule to make a pattern of numbers.
Each **input** or ***x*-value** has an **output** or ***y*-value** that fits the rule.
We can represent a function in a number of different ways:

1. The rule in words

Multiply the input by 3 and then subtract 2

2. Flow diagram



The 1st input is -2.
Use the rule:
$-2 \times 3 = -6$ then $-6 - 2 = -8$
So the output is -8.

3. Formula

$y = 3x - 2$

4. Table of values

x	– 2	– 1	0	1	2
y	– 8	– 5	– 2	1	4

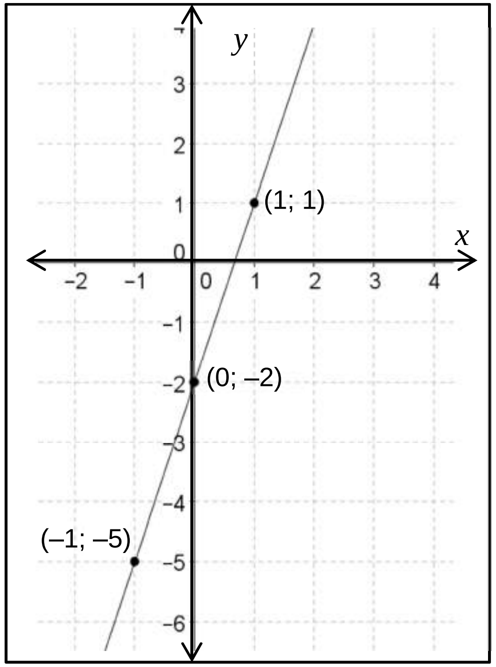
We can write the values from the table as **ordered pairs**.
The ***x*-coordinate** is always first,
the ***y*-coordinate** is always second.

$$(x;\ y)$$

The ordered pairs from the table are
(–2; –8) (–1; –5) (0; –2) (1; 1) (2; 4)

5. Graph

$y = 3x - 2$
Grade 8 & 9 only



The ordered pairs can be plotted on the **Cartesian plane.**
In this example, the points lie on a straight line and $x, y \in$ R, so we can join them with a straight line.

Grade 7, 8 and 9

If you need some revision on the Cartesian plane, see Unit 2.7 on graphs.

Worked examples:

1. Use the following rule: *To find the y-values, add 3 to the x-values and then divide by 5.*

   a) Complete the table of values.

x	–3	2	7		
y				3	4

   b) Write a formula for the rule.

.

Solutions

1. a)

x	– 3	2	7	**12**	**17**
y	**0**	**1**	**2**	3	4

The rules says add 3, then divide by 5 so when x = −3, we get −3 + 3 = 0 then 0 ÷ 5 = 0, So y = 0

To get from x to y we added 3 and then divided by 5, so to get from y back to x we must work backwards and use inverse operations i.e. we must multiply by 5 and subtract 3. So when y = 3: 3 × 5 = 15 and 15 – 3 = 12. So x = 12.

b) Formula: The rule is "*To find the y-values, add 3 to the x-values and then divide by 5.*" So $y = (x + 3) \div 5$ or $y = \frac{x+3}{5}$

2. Match the representation of the function in column 1 with the representation of the same function in column 2.

<table>
<tr><th>COLUMN 1</th><th>COLUMN 2</th></tr>
<tr><td>1. Subtract 1 from the input and then multiply by 4.</td><td>A
$y = 2x^2$</td></tr>
<tr><td>2.<table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>8</td><td>2</td><td>0</td><td>2</td><td>8</td></tr></table></td><td>B.
$y = 2x + 8$</td></tr>
<tr><td>3.
$y = \frac{x-1}{2}$</td><td>C.
Inputs: 2, 1, – 3 → $-1 \rightarrow \times 4$ → Outputs: 4, 0, –16</td></tr>
<tr><td>4.
Inputs: 0, $\frac{1}{2}$, 2 → $\times 2 \rightarrow +8$ → Outputs: 8, 9, 12</td><td>D.
Subtract 1 from the input and divide by 2.</td></tr>
</table>

Solutions:

2. 1C; 2A; 3D; 4B

Grade 7, 8 and 9

Questions

Unit 2.6: Functions

Grade 7, 8 and 9

1. For each flow diagram below, choose an input number and put it through the process in the flow diagram to determine the output number.

a) $\longrightarrow \boxed{\times 2} \longrightarrow$

b) $\longrightarrow \boxed{\times 2} \longrightarrow \boxed{-1} \longrightarrow$

2. For each of the word descriptions below, draw the flow diagram.

a) Multiply by –1.
b) Subtract 5 and then multiply by 2.
c) Divide by 2 and add 6.
d) Multiply by three and add 4.

3. For each flow diagram in Question 2, choose three different input values and calculate the corresponding output values.

4. For each question below, translate the sentences into formulae. Let the input value be x and the output value y in each case.

a) Add 6 to the input value.
b) To calculate the output value, multiply each input value by –4 and subtract 2.
c) Divide the input value by 2 and add 3 to calculate the output value.
d) Multiply the input value by $\frac{1}{2}$.
e) Add 6 to the input value that has been multiplied by –4.
f) Multiply the input value by –2 and then divide it by 3.
g) Subtract 8 from the input value that has been multiplied by 5.
h) The output value is equal to the input value.

5. For each formula that you wrote down in Question 4, complete a table like this:

x	–2	–1	0	1	2
y					

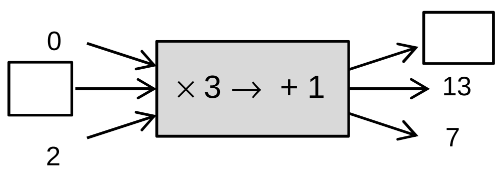
6. For each equation given below, draw up a table of values like this:

x	–1	0	1
y			

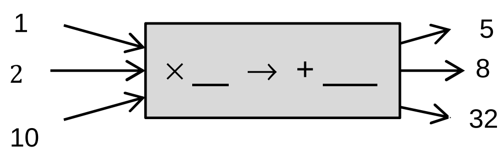
a) $y = x + 1$
b) $y = 2x - 3$
c) $y = \frac{1}{2}x - 1$
d) $y = -x + 1$
e) $y = -2x - 3$
f) $y = -\frac{1}{2}x - 1$

7. Fill in the missing values on the following flow diagrams:

a)



b)



Grade 8 & 9 only

8. A set of rectangles all have a perimeter of 24 units. The breadth of each rectangle (y) varies in relation to the length (x) using the formula $2(x + y) = 24$. Complete the table of values to represent this situation.

x	1	2	3	4	6					
y						5	4	3	2	1

9. The formula $b = 180° - \frac{360°}{n}$ gives the size b of each interior angle for a regular polygon with n sides.

a) Complete the table below.

number of sides (n)	3	4	5	6	10
angle size (b)					

b) What is the size of each interior angle of a regular polygon with 20 sides, and a regular polygon with 120 sides?

c) If each interior angle of a polygon is 150°, how many sides does it have?

10. The formula $y = 1{,}14x$ is used to calculate the price y of goods including VAT in rands, where x is the price in rands before VAT.

a) How much will you pay at the counter for goods that cost R38,00 without VAT?

b) How much will you pay for goods that cost R50,00 without VAT?
You may use a calculator.

c) x is the price in rands before VAT. Complete the table for the prices that include VAT.

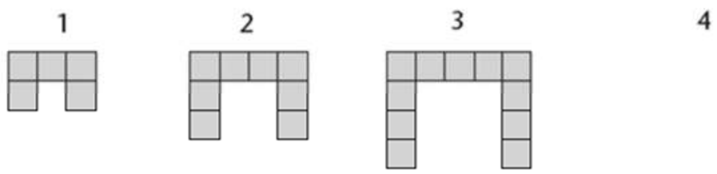
x	1	2	3	4	5	6	7	8	9
y									

d) An article costs R11,40 with VAT included. What is the price before VAT was added?

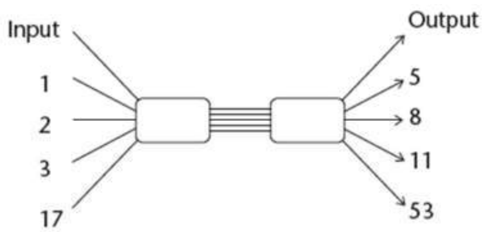
e) An article costs R342 with VAT included. What is the price before VAT was added?

11. Look at the tile pattern below.

a) Draw the 4th tile pattern. 12.



b) Complete the flow diagram below with the missing 'rule' to calculate the number of tiles in any step of the pattern.



Grade 9 only

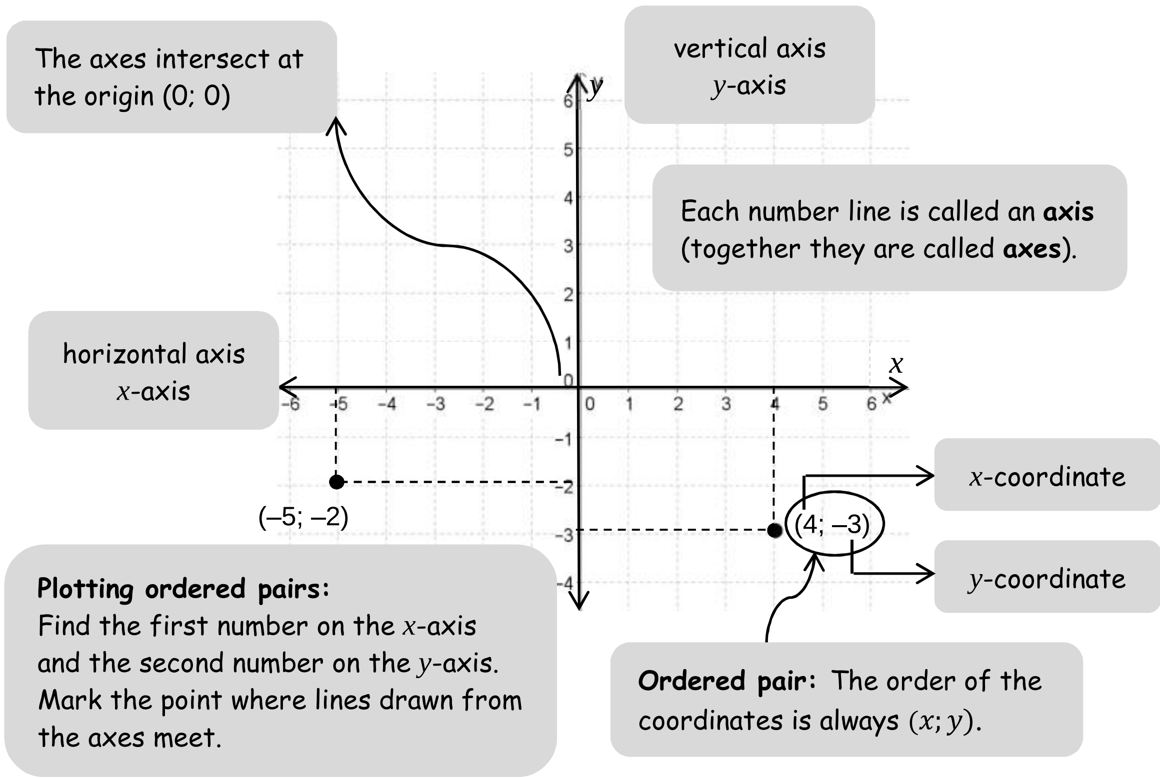
12. Refer to Question 6. For each equation, draw a Cartesian Plane. The x-axis must extend from –2 to 2, the y-axis from –5 to 5. You will have six Cartesian Planes, one for each equation. Use the tables to represent each equation as a graph on the separate axes.

Unit 2.7: Graphs

Grade 8 and 9

The Cartesian plane and ordered pairs:

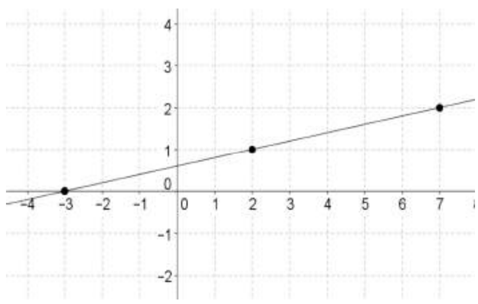
The Cartesian plane is made of two number lines perpendicular to each other.



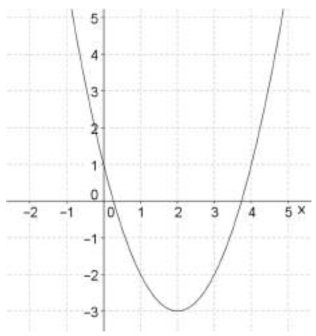
Grade 7, 8 and 9

Trends and features of graphs:

Any straight line graph is called a **linear** graph.
Any graph that is curved or changes direction in some way is **non-linear**.



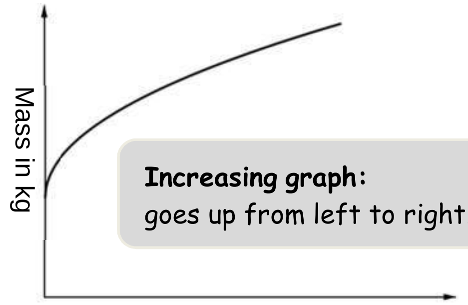
Linear graph



Non-linear graph

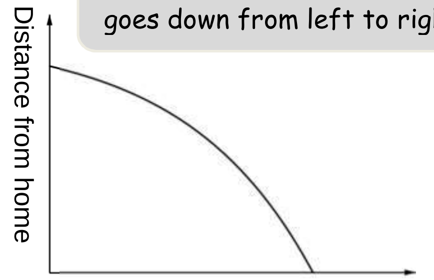
Grade 7, 8 and 9

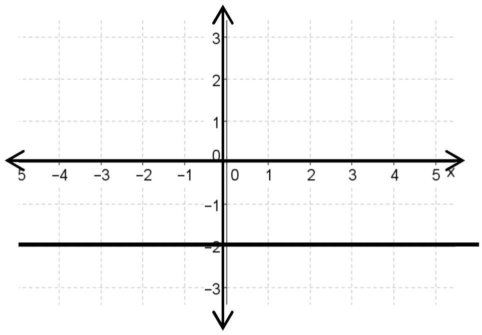
Constant, increasing or decreasing graphs:



Increasing graph:
goes up from left to right.

Decreasing graph:
goes down from left to right.





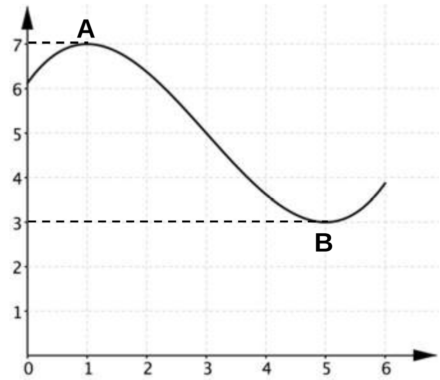
Constant graph:
The graph has a constant y-value, so it doesn't increase or decrease.

Maximum or minimum:

The maximum is the largest y-value that the graph reaches. The minimum is the smallest y-value that the graph reaches.

Maximum value of 7 at A when $x = 1$

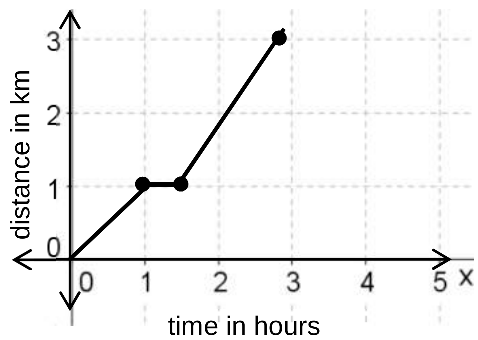
Minimum value of 3 at B when $x = 5$



Grade 8 and 9

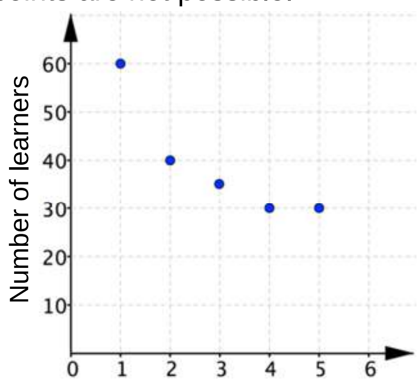
Continuous or discrete:

Continuous data can be joined because the values between data points are possible.



The journey of someone who runs at a constant speed for one hour, rests for half an hour and then runs at a constant speed for $1\frac{1}{2}$ hours.

Discrete data has data points that cannot be joined because the values between the points are not possible.



Number of learners attending extra mathematics classes in one school week

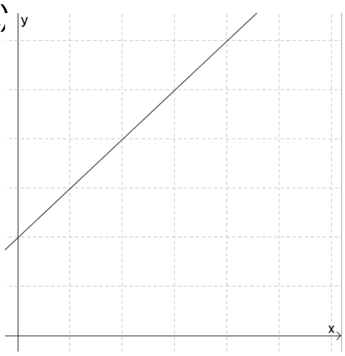
Refer to data handling for more about data.

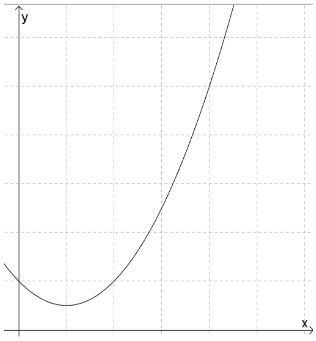
Questions

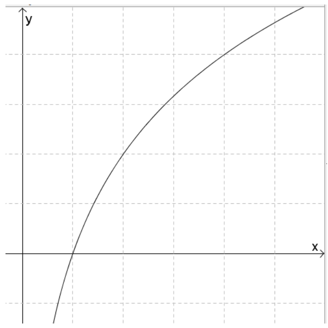
Unit 2.7: Graphs

Grade 7, 8 and 9

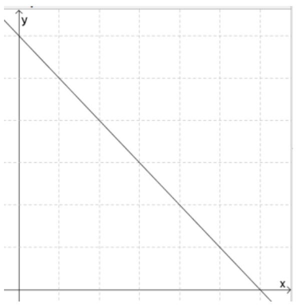
1. State whether the following graphs are linear or non-linear:

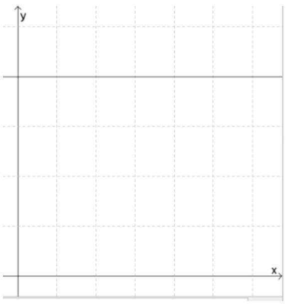
a)


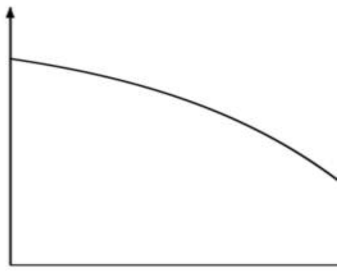
b)


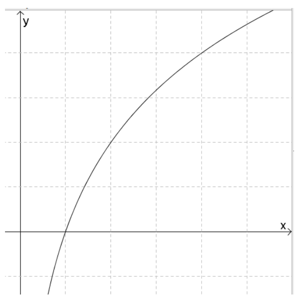
c)


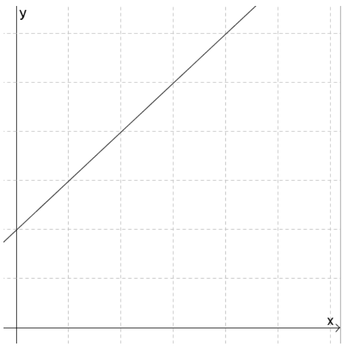
2. State whether the following lines are increasing, decreasing or constant:

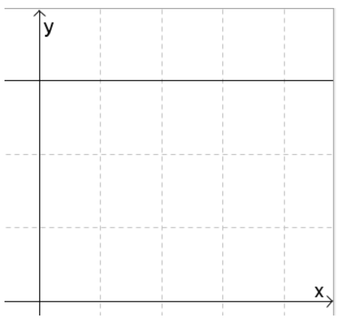
a)


b)


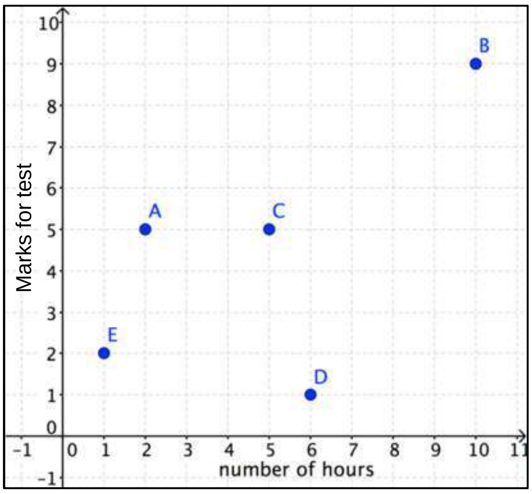
c)


d)


e)


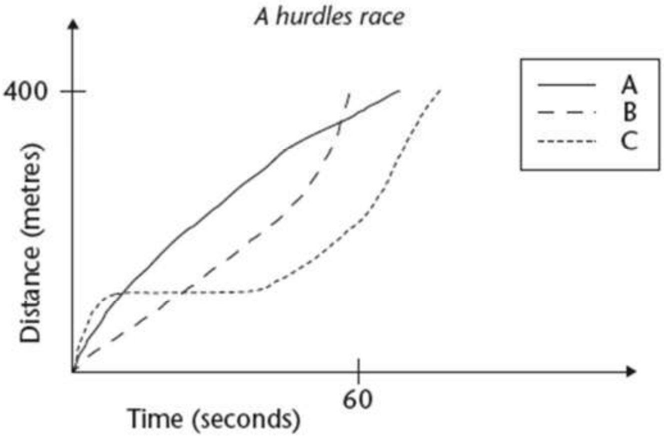
f)


3. A teacher records the number of hours that 5 learners (**A**nna, **B**ob, **C**am, **D**ineo, **E**brahim) study for a test and then records the mark they get for the test (out of 10). She plots the results:

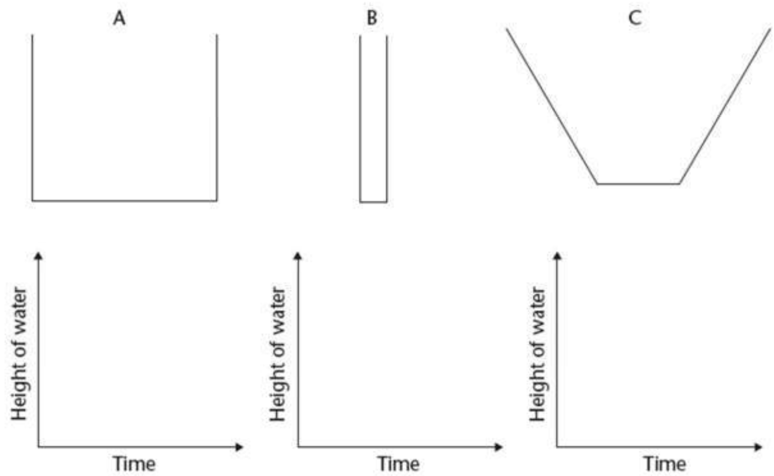


a) Which learner got the highest mark for the test?
b) Which learner got the lowest mark for the test?
c) How many hours did Anna study for and what mark did she get?
d) Which learners studied for more hours than Cam?
e) Which learners got more marks than Ebrahim?

4. The graph below shows the distance that three athletes, A, B and C, covered in a hurdles race in a certain time. Describe what happened during the race.



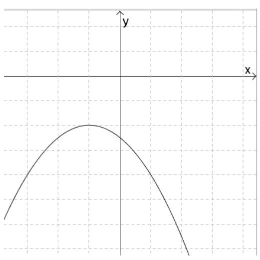
5. Water is dripping at a constant rate into three containers A, B and C at the same time. Draw graphs to show how the height of the water in each container changes over time.



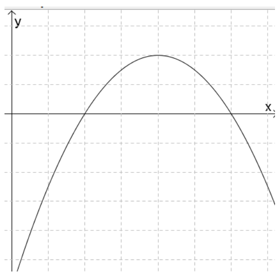
Grade 8 and 9

6. State whether the following graphs have a maximum value or a minimum value:

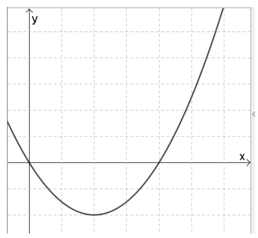
a)



b)



c)



7. Draw a Cartesian plane with x values and y values that are between –6 and 6. Plot the following ordered pairs on the Cartesian plane: A(5; 3) B(–2; –1) C(4; 1) D(–3; 4) E(0; 2)

8. a) Complete the table below:

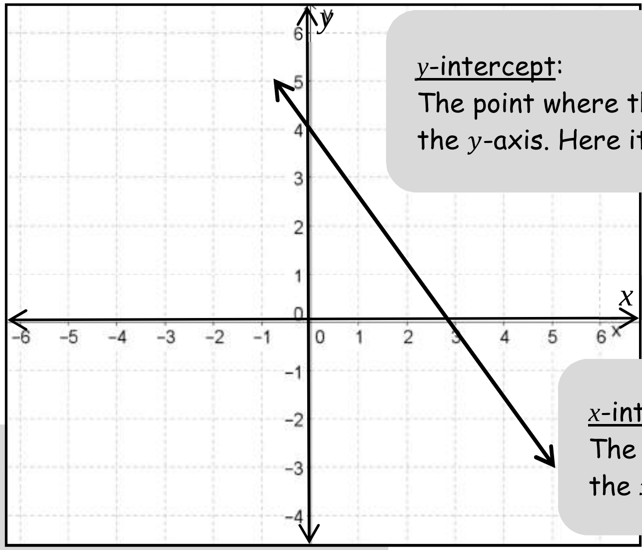
x	–3	–2	–1	0	1	2	3
$y = x^2 - 2$	7				–1		

b) Plot the points on the Cartesian plane.
c) Join the points to form a smooth curve.
d) What is the minimum value of the curve you have drawn?

Unit 2.8: Straight line graphs

Straight line graphs: Gradient and intercepts

- The **intercepts** are the points where a graph cuts (intersects with) the x-axis and the y-axis.
- The slope of the graph is called the **gradient**. The gradient (m) measures the steepness of a line. The gradient tells us how much we go up or down (the change in y) for each step we go along (the change in x).



y-intercept:
The point where the graph crosses the y-axis. Here it is (0; 4).

x-intercept:
The point where the graph crosses the x-axis. Here it is (3; 0).

Calculating the gradient:

$$\text{Gradient } (m) = \frac{\text{change in } y}{\text{change in } x}$$

So for any two points on the line, point 1: $(x_1;\ y_1)$ and point 2: $(x_2;\ y_2)$,

gradient $(m) = \frac{y_2 - y_1}{x_2 - x_1}$

Here if we take point 1 (3; 0) and point 2 (0; 4), then $m = \frac{4-0}{0-3} = \frac{4}{-3}$

If the gradient $m > 0$, then the graph is increasing.
If the gradient $m < 0$, then the graph is decreasing.
If the gradient m = 0, then the graph is horizontal

Grade 9

Worked examples:

Determine the gradient of the following lines and state whether the lines are increasing, decreasing or constant.

1. The line passing through the points $(2;1)$ and $(4;3)$

   Solution: $m = \frac{3-1}{4-2} = \frac{2}{2} = 1$

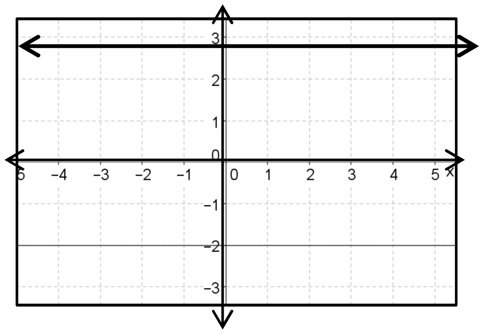
   The line is increasing because the gradient is positive.

2. The line passing through the points $(-1;3)$ and $(-3;3)$.

   Solution:

   $m = \frac{3-3}{-3-(-1)} = \frac{0}{-3+1} = \frac{0}{-2} = 0$

   The line is constant because the gradient is 0.



The standard equation for a straight line graph:

$y = mx + c$ where m is the gradient and c is the y-coordinate of the y-intercept.

Worked examples:

Write the equation in standard form and state the gradient and y-intercept:

1. $y - 3x + 4 = 0$

   $y = 3x - 4$

   $\therefore$ gradient is 3 and the y intercept is $(0;-4)$

2. $2x - 2y = 3$

   $2x - 3 = 2y$

   $x - \frac{3}{2} = y$

   $y = x - \frac{3}{2}$ (It is easier to keep y on the left)

   $\therefore$ gradient is 1 (from $1x$) and the y intercept is $\left(0;-\frac{3}{2}\right)$

Grade 9

Drawing a straight line graph from the equation:

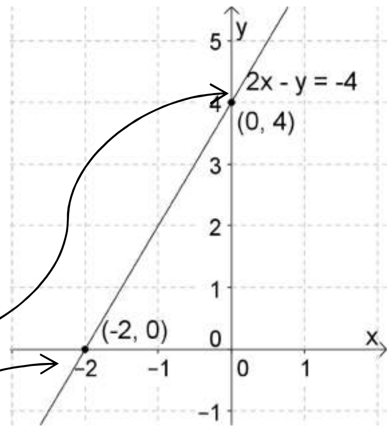
1. Calculate the intercepts:

Worked example: Draw the graph of $2x - y + 4 = 0$

- ✓ Find the y-intercept by putting x = 0
- ✓ Find the x-intercept by putting y = 0

x intercept:
When $y = 0$,
$2x - (0) + 4 = 0$
$2x = -4$
$x = -2$
Point to plot: $(-2; 0)$

y intercept
When $x = 0$,
$2(0) - y + 4 = 0$
$-y + 4 = 0$
$4 = y$
Point to plot: $(0; 4)$



Join the two intercepts.

2. Use any two points:

Worked example:
Draw the graph for $y = 2x$

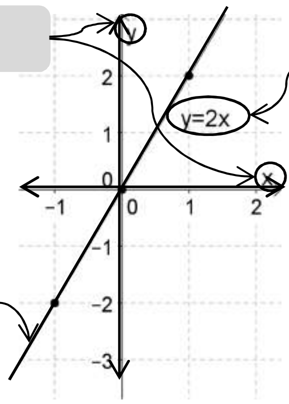
- ✓ Use any two input or x-values. It is easy to use values close to 0 eg –1 and 1
- ✓ Substitute them into the equation to find the output values.
- ✓ You can check your calculations by using a third x-value and y-value.

x	–1	0	1
y	–2	0	2

Plot the points

Label the axes

Label the graph with its equation.



Join the points and extend the line past the end points.

Grade 9

Horizontal and vertical graphs:

- ✓ Horizontal graphs: $y = k$, where k is a number (constant)
- ✓ Vertical graphs: $x = k$, where k is a number (constant)

Worked example:

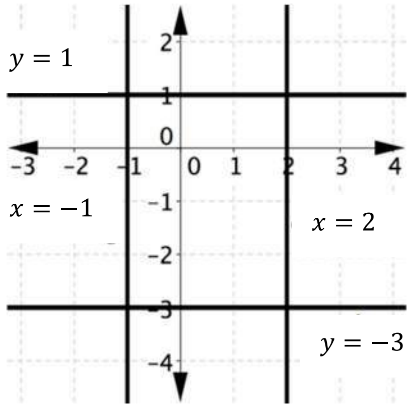
Draw the following graphs on the same set of axes:

$x = 2$

$y = -3$

$x = -1$

$y = 1$



Finding the equation of a graph:

To find the equation of a graph:

- ✓ calculate the gradient (m)
- ✓ find the y intercept (c)
- ✓ substitute the m and c values into the standard equation $y = mx + c$

Worked examples:

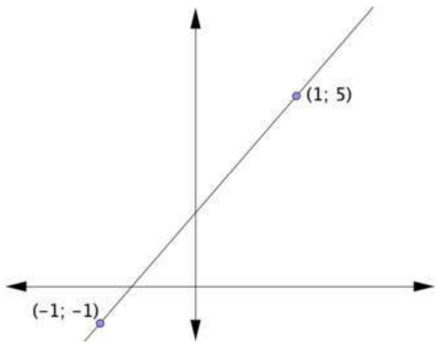
1. Determine the equation of the line passing through $(0; 4)$ and $(-1; 5)$

   Step 1: Use the points to find the gradient: $m = \frac{5-4}{-1-0} = \frac{1}{-1} = -1$

   Step 2: Find the y value of the y intercept.
   $(0; 4)$ is the y intercept.

   ∴ Equation is $y = -x + 4$

2. Determine the equation of the graph:



   Step 1: Use the points to find the gradient: $m = \frac{5-(-1)}{1-(-1)} = \frac{6}{2} = 3$

   Step 2: Substitute either point into
   $y = 3x + c$ to calculate c.
   $5 = 3(1) + c$ using the point (1; 5)
   so $c = 5 - 3 = 2$
   So the equation is $y = 3x + 2$

Grade 9

Questions

Unit 2.8: Straight line graphs

Grade 9 only

1. Marco fills a $20\ l$ bucket of water from a tap. He records the following data:

Time (in seconds)	0	10	20	30	40	50
Volume (in litres)	0	4	8	12	16	20

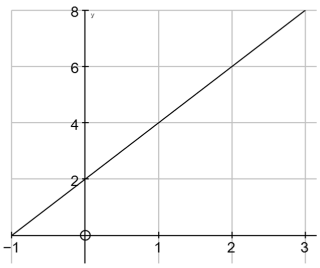
   a) Draw a graph to represent this data.
   b) Can you extend this graph past the end points of the data? Explain.

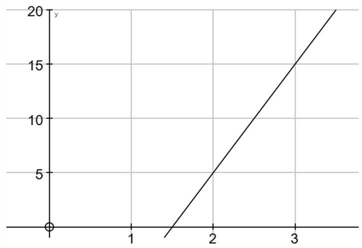
2. Lindiwe measures her distance run over time. She records this data:

Time (in minutes)	0	2	4	6	8	10
Distance (in metres)	0	350	700	1 050	1 400	1 750

   a) Draw a graph to represent this data.
   b) Do you think Lindiwe's distance will continue to increase at this rate if she runs for longer?
   c) What do you think will happen to the steepness of the graph as Lindiwe continues to run?

3. Calculate the gradient of the lines passing through:
   a) (1; 1) and (5; 3) b) (0; 2) and (5; 3)
   c) (–5; –2) and (–1; 2) d) (–2; 3) and (1; 0)

4. Calculate the gradient of the following lines:
   a)


   b)


5. For the following equations,
   i) Rewrite each equation below in standard form.
   ii) State the gradient and y-intercept of each line *(you do not need to draw the graphs)*

   a) $x + y - 2 = 0$ b) $y - x + 2 = 0$
   c) $3x - 5y = -15$ d) $2y - 2 = -2x$

6. Draw the following graphs on separate axes using a table of values:
   a) $y = 2x$ b) $y = -3x$
   c) $y = x$ d) $y = -\frac{1}{2}x$

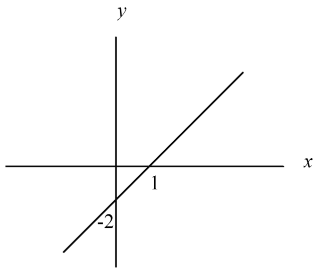
7. By first calculating the intercepts, draw the following graphs, each on a separate set of axes:
   a) $y = x - 4$ b) $y = -2x + 3$
   c) $2y = 2x + 8$ d) $2x - 6y = 12$
   e) $y = \frac{2}{3}x - 1$

8. Draw the following graphs accurately on the same set of axes:

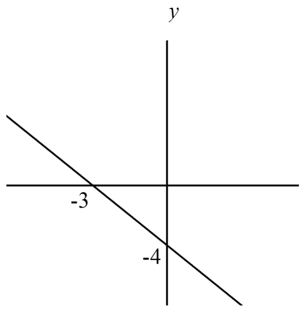
a) $y = 3x$ b) $y = -x + 4$ c) $x = -1$ iv) $y = 2$

9. Find the equations of the following lines:

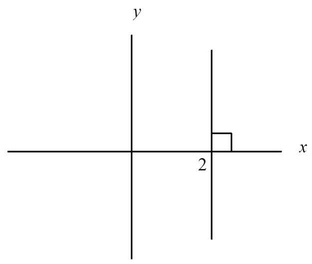
a)



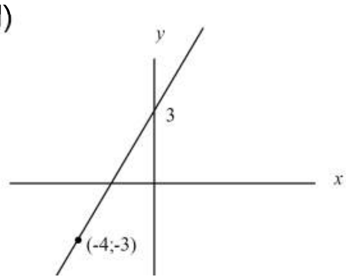
b)



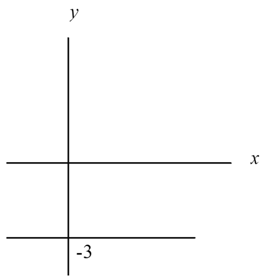
c)



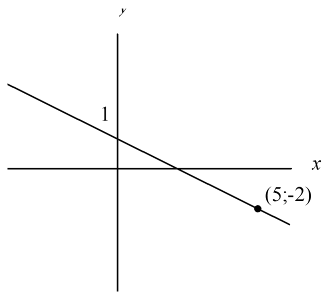
d)



e)



f)



10. For each of the tables of values below, decide if the points, when plotted, will lie on a straight line. If they do, then find the equation of the straight line.

a)

x	–1	0	1	2
y	3	5	7	9

b)

x	–3	1	4	8
y	2	4	6	8

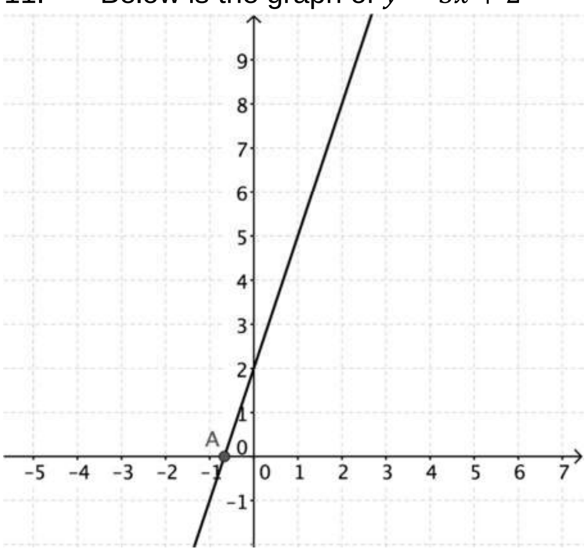
c)

x	–1	0	1	2
y	2	5	9	14

d)

x	–1	0	1	2
y	9	6	3	0

11. Below is the graph of $y = 3x + 2$



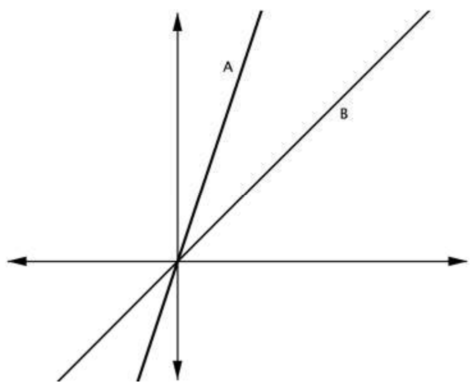
a) From the graph read off the value of x for which $3x + 2 = 8$.

b) Solve the equation $3x + 2 = 8$ and confirm you get the same answer as for a).

c) Find the coordinates of A.

12. One of the graphs A and B shows $y = 3x$ and the other shows $y = x$.

Identify the equation for graph A and the equation for graph B.



Unit 3.1: Straight line Geometry

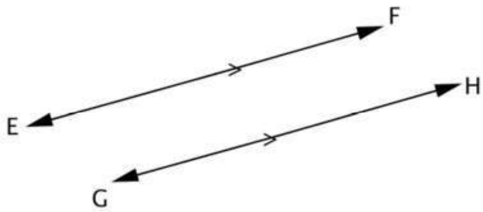
Line
A line is a set of points that goes on and on in both directions.

Ray
A ray is a set of points with a definite starting point but carries on forever in one direction so there is no end point.

Line segment
A line segment is a set of points with a definite starting point and a definite end point.

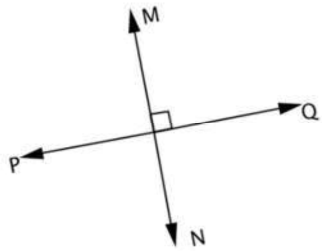
Parallel lines stay the same distance apart.

We say EF || GH



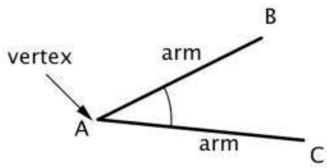
Perpendicular lines cross each other at right angles (90°).

We say MN ⊥ PQ

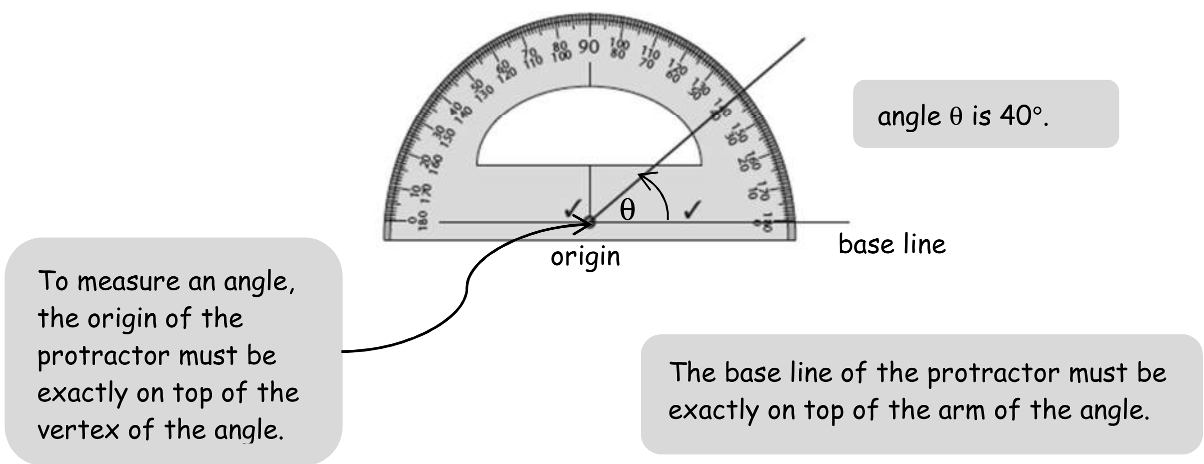


Angles
An **angle** is formed between two rays or line segments that meet at a vertex. We say the angle has two arms.

We call this angle $B\hat{A}C$



We measure angles with a PROTRACTOR.

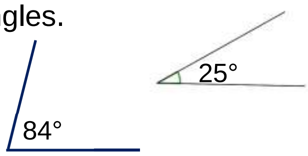


Angles

Grade 7, 8 and 9

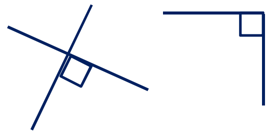
Acute angles

Angles between 0° and 90° are called acute angles.
So 25°, 84° and 5° are all acute angles.



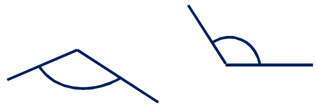
Right angles

Angles of exactly 90° are called right angles.



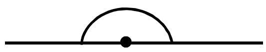
Obtuse angles

Angles between 90° and 180°.
So 91°, 123° and 176° are all obtuse angles.



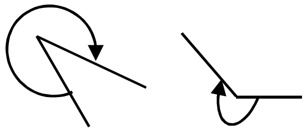
Straight angles

Angles of exactly 180° are called straight angles. The arms make a straight line.



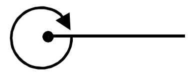
Reflex angles

Angles between 180° and 360° are called reflex angles.
So 190°, 270° and 312° are reflex.

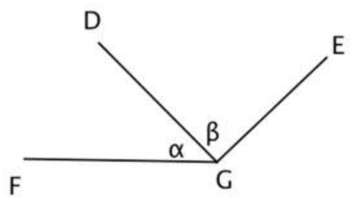


Revolution

An angle of 360° is called a revolution. The angle makes a full circle around a point.



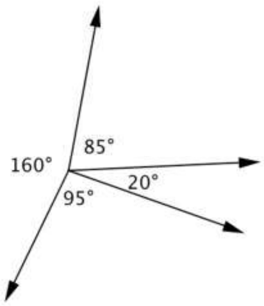
Adjacent angles share a common vertex and a common arm and are on either side of the common arm.



$F\hat{G}D$ (marked α) and $D\hat{G}E$ (marked β) are adjacent angles.
They share G as a vertex and have a common arm DG.

Grade 8 and 9

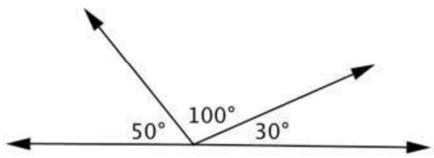
Angles around a point add up to 360°
(we say that 360° is a **revolution**)



$160^o + 85^o + 20^o + 95^o = 360^o$

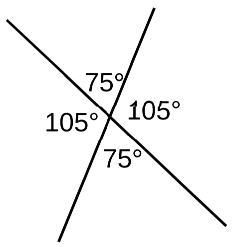
Angles on a **straight line** add up to **180°**.

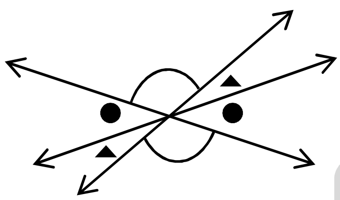
Angles that add up to 180° are called **supplementary** angles.



$50^o + 100^o + 30^o = 180^o$

When two lines or line segments intersect they create two pairs of equal **vertically opposite angles.**





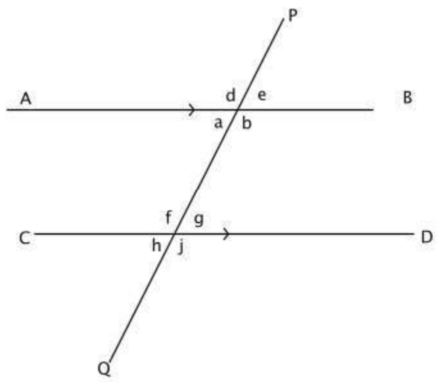
Intersect:
Cut each other at a point.

To find vertically opposite angles:
Look for an X-shape.

Equal angles on parallel lines

A line which cuts (intersects) a pair of parallel lines is called a **transversal**.
The transversal creates many angles with the two parallel lines.

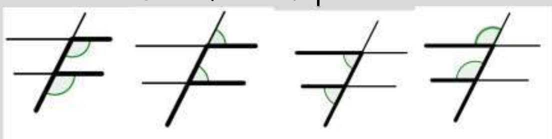
AB || CD and PQ is a transversal.



- $\hat{e} = \hat{g}$, $\hat{d} = \hat{f}$, $\hat{a} = \hat{h}$ and $\hat{b} = \hat{j}$
  (corres ∠s AB || CD)
- $\hat{a} + \hat{f} = 180°$ and $\hat{b} + \hat{g} = 180°$
  (coint ∠s AB || CD)
- $\hat{a} = \hat{g}$ and $\hat{b} = \hat{f}$
  (alt ∠s AB || CD)

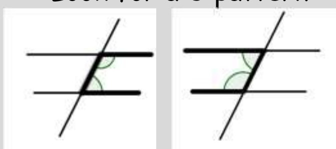
Corresponding angles on parallel lines are equal.

Look for an F pattern:



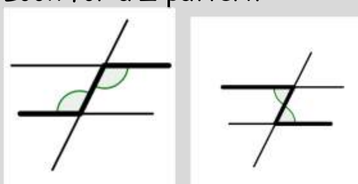
Co-interior angles on parallel lines add up to 180°.

Look for a U pattern:



Alternate angles on parallel lines are equal.

Look for a Z pattern:



Grade 8 and 9

Questions

Unit 3.1 Straight line geometry

Grade 7, 8 and 9

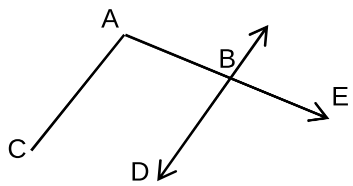
1. What do we call
   a) lines that are equidistant from each other?
   b) lines that intersect at right angles?
   c) the point where the two arms of an angle meet?

2. 

Is PQ a line, a ray or a line segment?

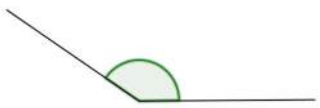
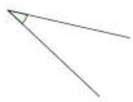
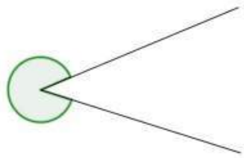
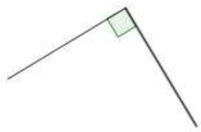
3. Which of these is a line segment?

4. In the diagram, identify and name: a) a line b) a line segment c) a ray



5. How many degrees are there in a revolution?
6. What do we call angles that share a common vertex and a common arm?
7. Give the name of the type of angle for each of the angles below:
   a) 40° b) 180° c) 275° d) 120°

   e) f) g) h)

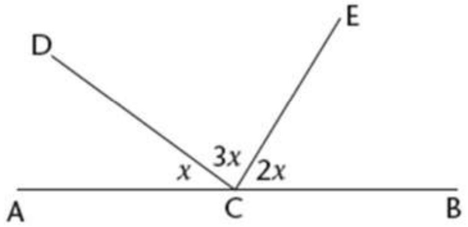
8. An angle that is between 90° and 180° is called an ________________ angle.
9. A reflex angle lies between ____° and ____°.
10. A straight angle has an angle of _____° and is half a _____________________.
11. A right angle is _______°.
12. The vertex of an angle is _______________________.
13. Parallel lines ______________________________.
14. Perpendicular lines intersect at ________________.
15. ⊥ is the symbol for _____________ lines.
16. ∥ represents ________________lines.

Grade 8 and 9 only

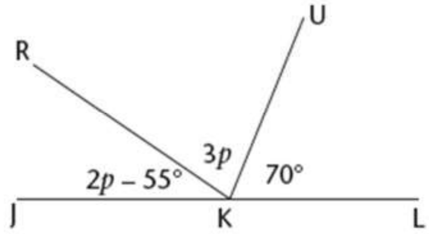
1. True or false?

   Two adjacent acute angles will always form an obtuse angle.

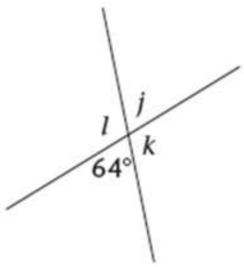
2. Calculate the size of:
   a) x
   b) $E\hat{C}B$



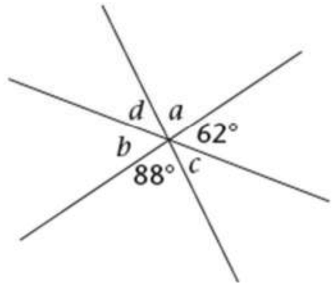
3. Calculate the size of:
   a) p
   b) $J\hat{K}R$



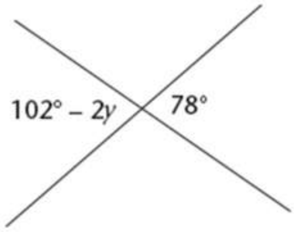
4. Calculate j, k and l.



5. Calculate a, b, c and d.

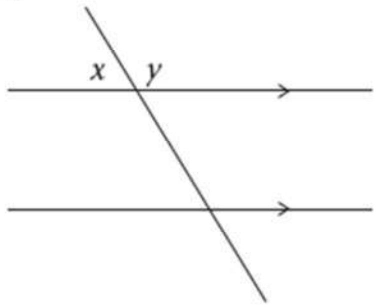


6. Calculate the value of y.

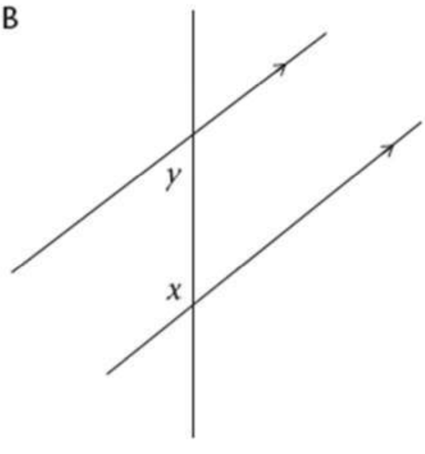


7. Copy the diagrams below. Without measuring, mark all angles equal to x and all angles equal to y. Give reasons.

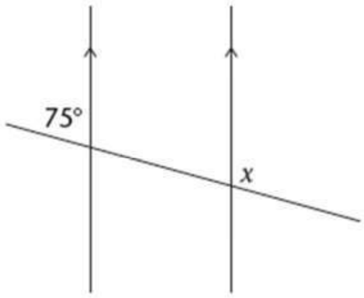
A

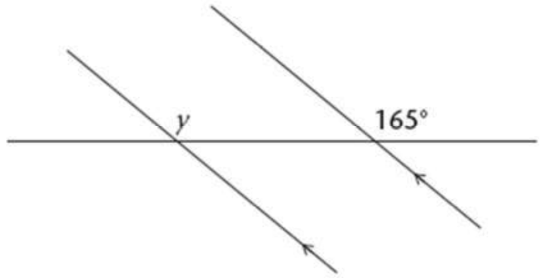


B

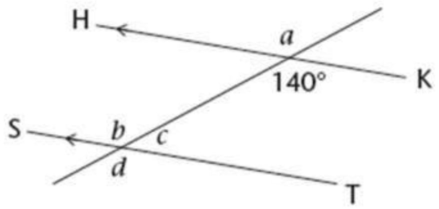


8. Give the values of x and y below.

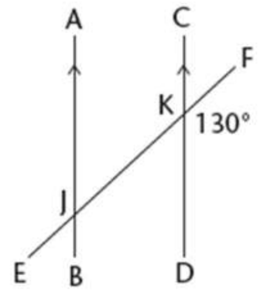




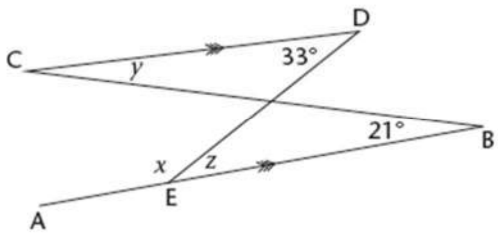
9. Calculate the values of b, c and d.



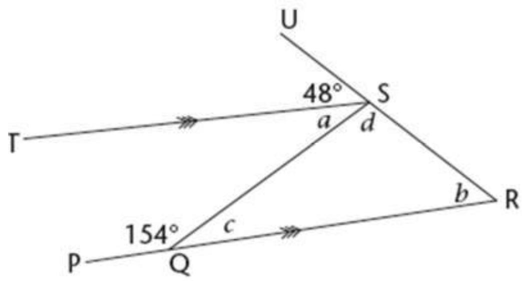
10. Find the sizes of $A\hat{J}F$ and $C\hat{K}E$ in the diagram below.



11. Calculate the sizes of x, y and z.

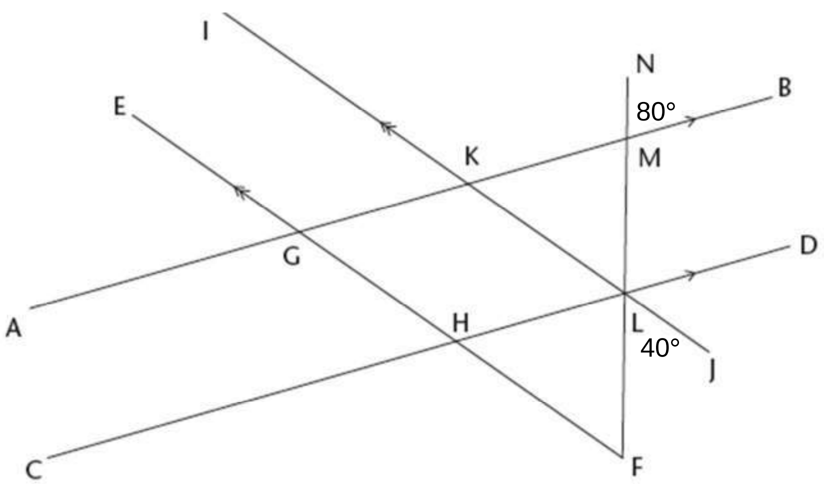


12. Calculate the sizes of a, b, c and d.



Grade 9 only

13. In the diagram, AB and CD are parallel. EF and IJ are also parallel.
$N\hat{M}B$ = 80° and $J\hat{L}F$ = 40°. Find the sizes of as many angles as you can, giving reasons.



Unit 3.2: Triangles

Grade 8 & 9

Triangle:
A triangle is a closed shape with three sides and three interior angles.

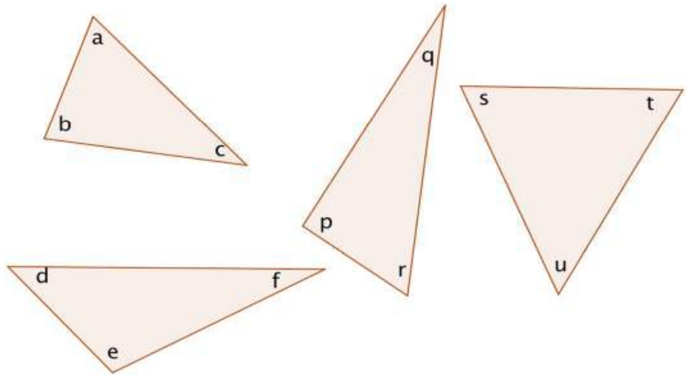
Important:
The **angles of a triangle** add up to 180°.

$a + b + c = 180°$

$d + e + f = 180°$

$p + q + r = 180°$

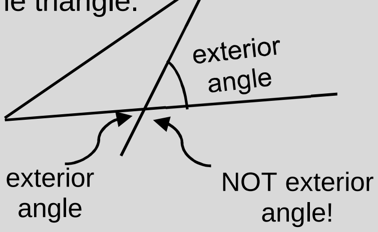
$s + t + u = 180°$



Grade 9

The exterior angle of a triangle

When one side of a triangle is made longer (extended), an **exterior angle** is created between the extended line and a side of the triangle.

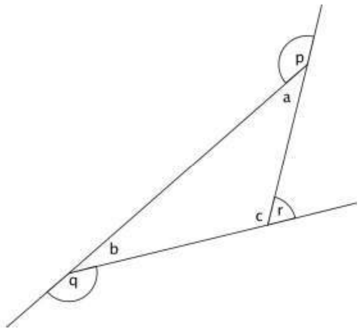


Important:
The **exterior angle of a triangle** is equal to the sum of the two interior opposite angles.

$r = a + b$

$q = a + c$

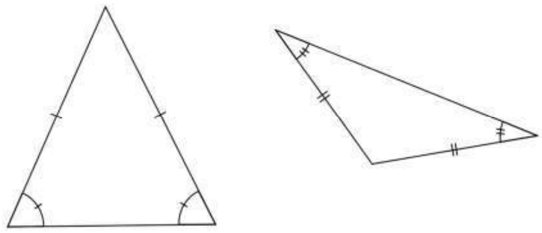
$p = b + c$



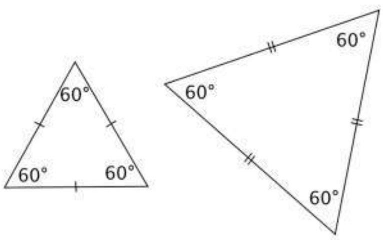
Grade 7, 8 & 9

Naming triangles according to their sides

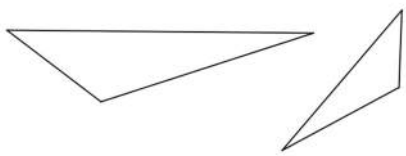
- An **isosceles triangle** has two equal sides.
- In an isosceles triangle, the angles opposite the equal sides are equal.



- An **equilateral triangle** has all three sides equal.
- All three angles are 60°.

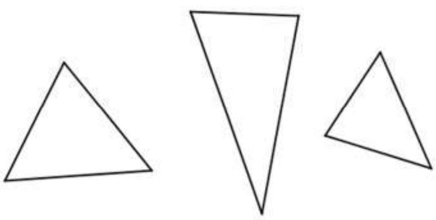


- A **scalene triangle** has all three sides different in length.
- Angles are all different sizes.

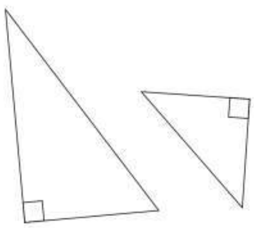


Naming triangles by their angles

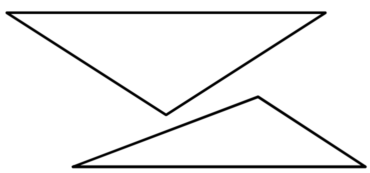
Acute-angled triangle:
All angles less than 90° (acute)



Right-angled triangle:
One 90° angle (right)



Obtuse-angled triangle:
One angle of the triangle is greater than 90° (obtuse)

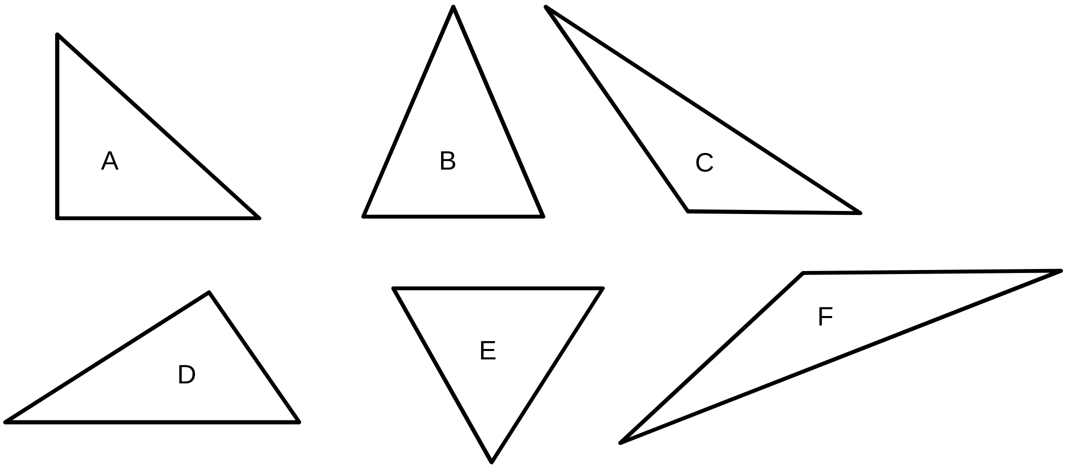


Questions

Unit 3.2: Triangles

Grade 7 only

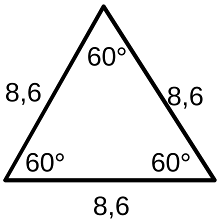
1. a) Measure the sides and angles of each triangle.
   b) Use your measurements to decide if each triangle is acute-angled, right-angled or obtuse-angled.
   c) Also decide if each triangle is isosceles, equilateral or scalene.



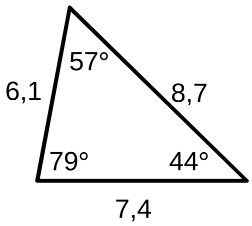
Grade 7, 8 and 9

2. Look at the measurements or markings given in the diagrams. Do not measure the angles or sides.
   - For each triangle, state whether it is acute-angled, right-angled or obtuse-angled
   - For each triangle, state whether it is isosceles, equilateral or scalene.

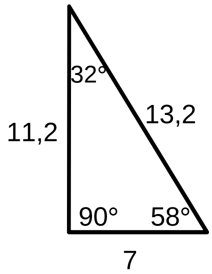
a)



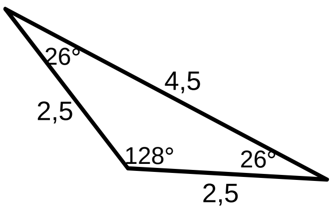
b)



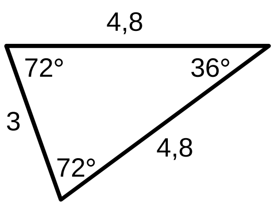
c)



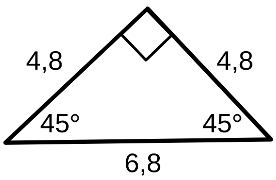
d)



e)

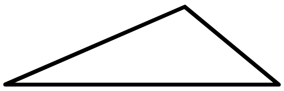


f)

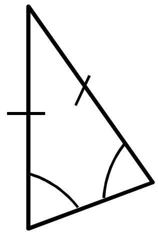


3. Name each triangle by its angles and its sides. Equal sides and angles are shown.

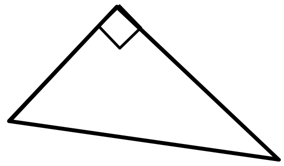
a)



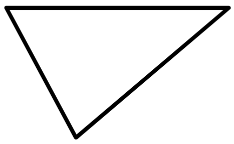
b)



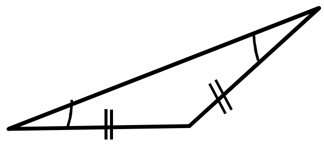
c)



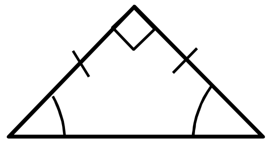
d)



e)



f)

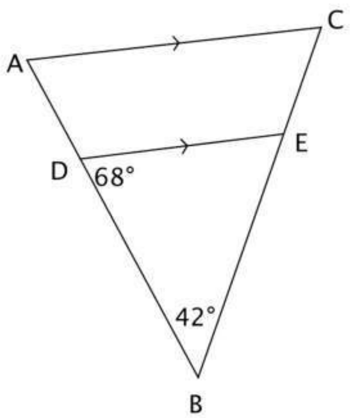


4. Draw an example of each type of triangle described. Mark the equal angles, equal sides and right angles. If no triangle can be drawn, write “not possible”.

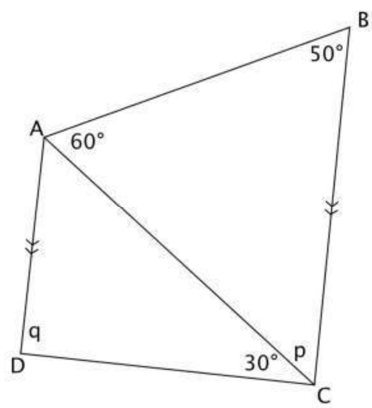
a) acute isosceles b) right scalene c) right isosceles
d) right equilateral e) acute scalene f) obtuse scalene
g) right obtuse h) equilateral

Grade 8 and 9

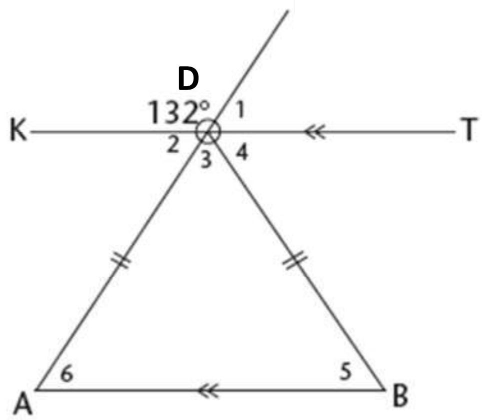
5. Find the size of $A\hat{C}B$.



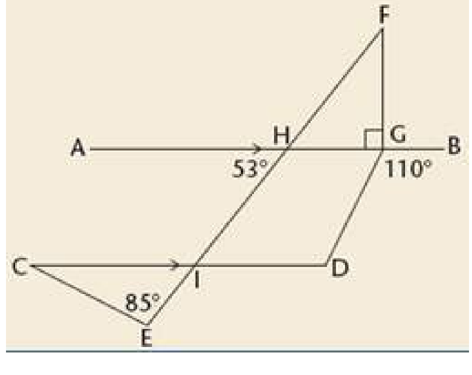
6. Find the sizes of the angles marked p and q.



7. Calculate the sizes of:
$\hat{D}_1$, $\hat{D}_2$, $\hat{D}_3$, $\hat{D}_4$, $\hat{B}_5$ and $\hat{A}_6$.



8. In the diagram, AB ∥ CD. Calculate the sizes of $F\hat{H}G$, $\hat{F}$, $\hat{C}$ and $\hat{D}$. Give reasons for your answers.

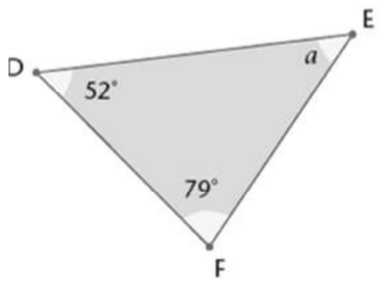


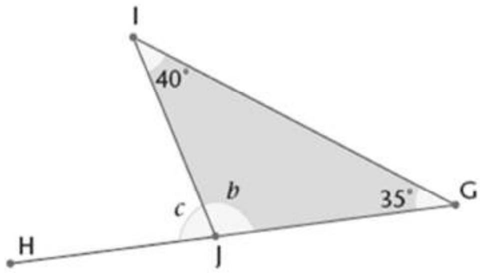
Grade 9

9. Read each statement carefully. Decide for which types of triangles the statement is true.

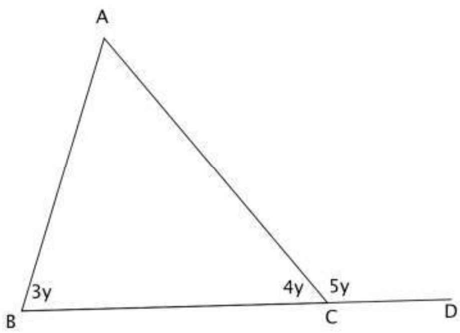
Statement	True for these triangles:
a) Two sides of the triangle are equal.	
b) One angle of the triangle is obtuse.	
c) Two angles of the triangle are equal	
d) All three angles of the triangle are equal to 60°.	
e) The exterior angle is equal to the sum of the opposite interior angles.	
f) The longest side of the triangle is opposite the biggest angle.	
g) The sum of the interior angles of the triangle is 180°.	

10. Calculate the sizes of the unknown angles.



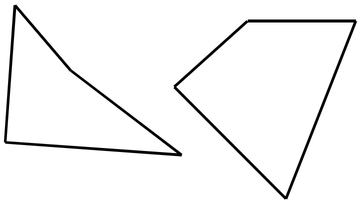


11. Find the size of $B\hat{A}C$.



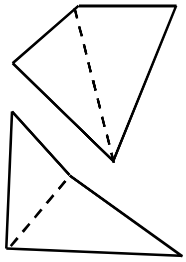
Unit 3.3: Quadrilaterals

A **quadrilateral** is a closed shape with four sides.



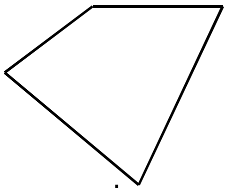
Divide the quadrilateral up into two triangles.
We know the angles of each triangle add up to 180°.

So the sum of the interior angles of a quadrilateral is 180° + 180° = 360°

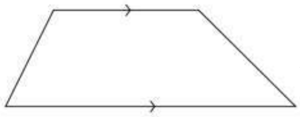


Definitions of quadrilaterals:

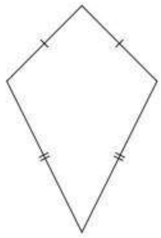
A **quadrilateral** is a closed figure with four sides.



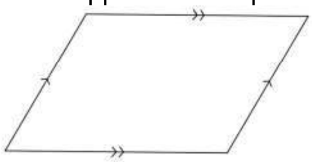
A **trapezium** is a quadrilateral with one pair of opposite sides parallel.



A **kite** is a quadrilateral with two pairs of adjacent sides equal.

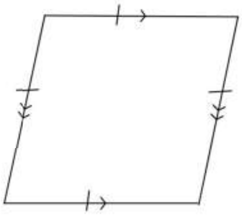


A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

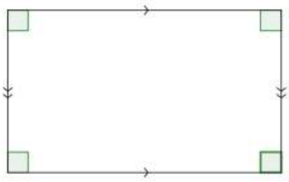


A **rhombus** is a parallelogram with all four sides equal.

A rhombus is a special kind of parallelogram.



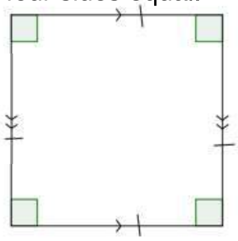
A **rectangle** is a parallelogram that has all angles equal to 90°.



A rectangle is a special kind of parallelogram.

A **square** is a rectangle with all four sides equal.

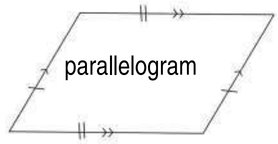
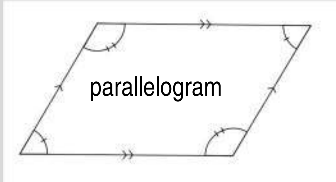
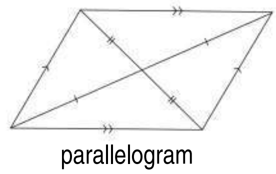
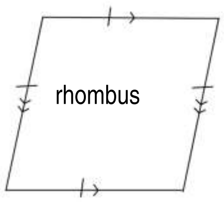
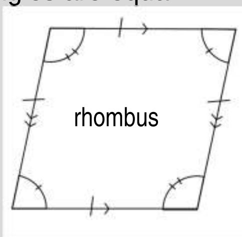
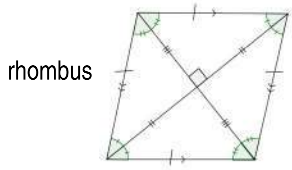
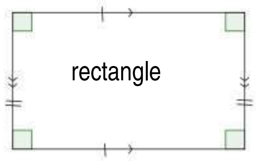
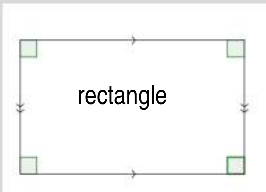
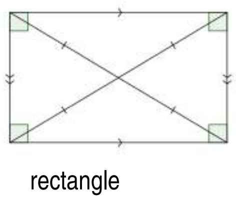
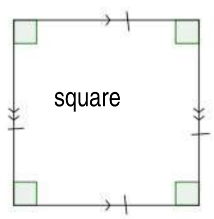
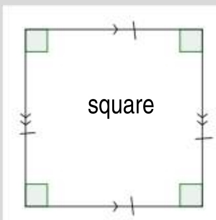
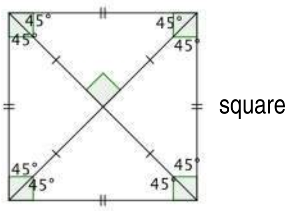
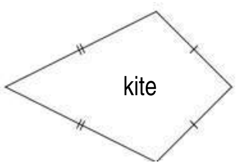
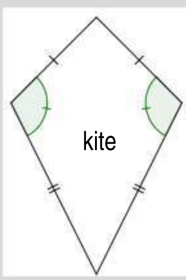
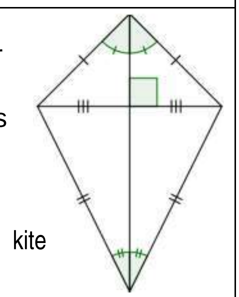
A square is a special kind of rectangle.



A square is also a special kind of rhombus.

Grade 7, 8 and 9

Properties of quadrilaterals

Sides Grade 7, 8, 9	**Angles Grade 8, 9**	**Diagonals Grade 9**
• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are equal. 	• Both pairs of opposite angles are equal. 	• Diagonals bisect each other. 
• Both pairs of opposite sides are parallel. • All four sides are equal. 	• Both pairs of opposite angles are equal. 	• Diagonals bisect each other at right angles. • Diagonals bisect the interior angles of the rhombus. 
• Both pairs of opposite sides are parallel. • Both pairs of opposite sides are equal. 	• All angles equal 90°. 	• Diagonals are equal and bisect each other. 
Both pairs of opposite sides are parallel. All four sides are equal. 	All angles equal 90°. 	• Diagonals are equal to each other. • Diagonals bisect each other at right angles. • Diagonals bisect the interior angles of the square. 
One pair of opposite sides are parallel. 		
Two pairs of adjacents sides equal. 	Opposite angles between unequal sides are equal. 	One diagonal of the kite bisects the other diagonal at 90°. One diagonal bisects the interior angles. 

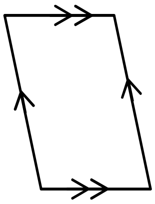
Questions

Unit 3.3: Quadrilaterals

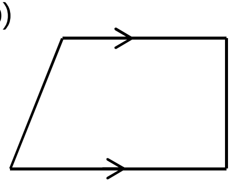
Grade 7, 8 and 9

1. Name each shape with the most specific name of the shape.

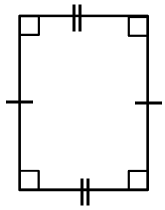
a)



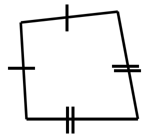
b)



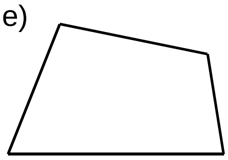
c)



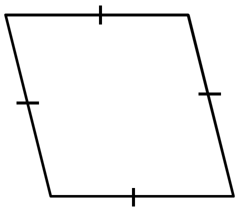
d)



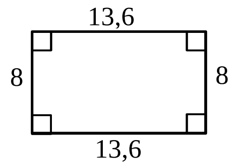
e)



f)



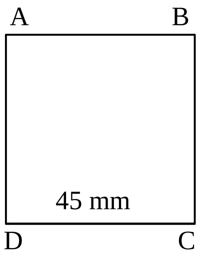
g)



2. Use the properties of quadrilaterals to find the size of the angles and lengths of the sides.

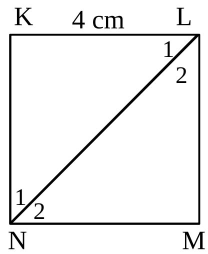
a) A square with side 45 mm.

AB = _____
BC = _____
AD = _____
$\hat{A}$ = _____
$\hat{B}$ = _____
$\hat{C}$ = _____
$\hat{D}$ = _____

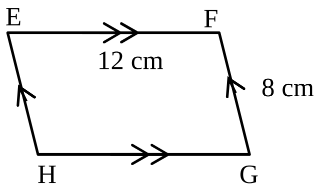


b) A square with side 4 cm.

KN = _____
NM = _____
$\hat{L}_1$ = _____
$\hat{L}_2$ = _____
$\hat{N}_1$ = _____
$\hat{N}_2$ = _____



c)

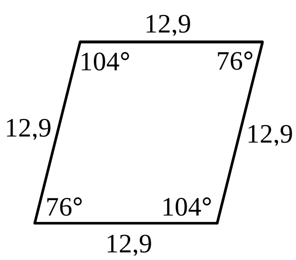


EH = _____
GH = _____

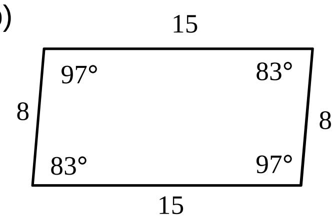
Grade 8 and 9

3. Name each shape with the most specific name of the shape.

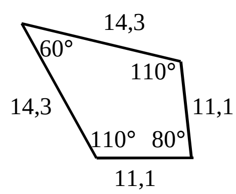
a)



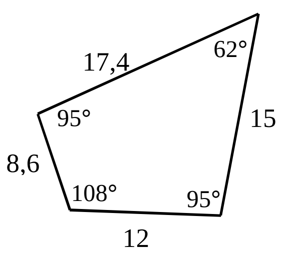
b)



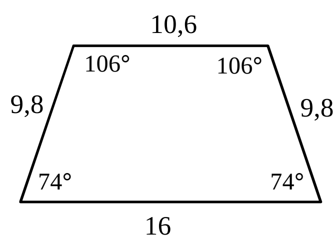
c)



d)

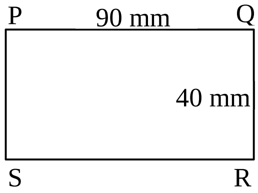


e)



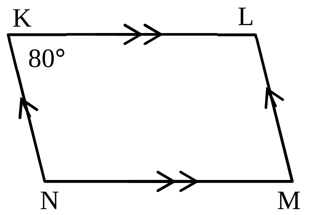
4. Use the properties of quadrilaterals to find the size of the angles and lengths of the sides.

a) PQRS is a rectangle.



PS = _____
RS = _____
$\hat{P}$ = _____
$\hat{Q}$ = _____
$\hat{R}$ = _____
$\hat{S}$ = _____

b) KLMN is a parallelogram.



$\hat{L}$ = _____
$\hat{M}$ = _____
$\hat{N}$ = _____

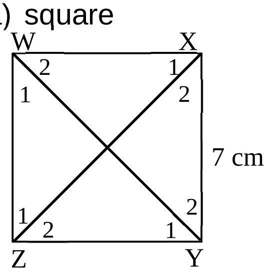
Grade 9

5. The following table is a summary of the properties of diagonals of quadrilaterals. Complete the table with ticks for the properties of each type of quadrilateral.

	parallelogram	rectangle	square	rhombus	trapezium	kite
diagonals bisect each other						
diagonals cut at right angles						

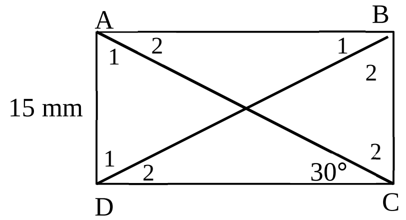
6. Use the properties of quadrilaterals to find the size of the angles and lengths of the sides.

a) square



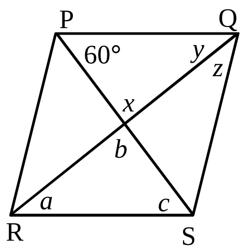
WX = _____
WZ = _____
$\hat{X}_1$ = _____
$\hat{X}_2$ = _____
$\hat{Y}_1$ = _____
$\hat{Z}_2$ = _____

b) rectangle



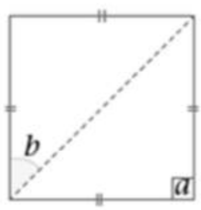
BC = _____
$\hat{C}_2$ = _____
$\hat{A}_1$ = _____
$\hat{A}_2$ = _____

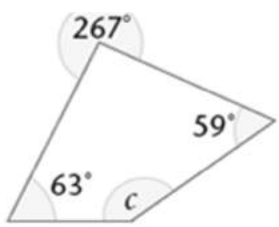
c) rhombus

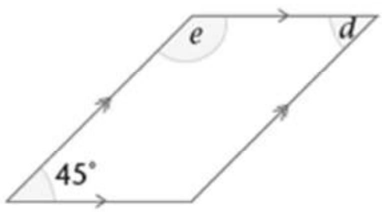


x = _____
y = _____
z = _____
a = _____
b = _____
c = _____

7. Determine the sizes of angles a to e in the quadrilaterals below. Give reasons for your answers.







Unit 3.4: Congruence and similarity

Similar shapes

Two figures are **similar** when they have exactly the same shape, but they can be different sizes.

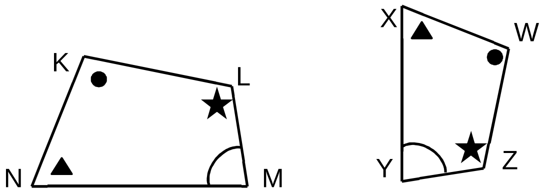
Because they have the same shape, similar figures have

- corresponding angles equal, and
- corresponding sides in the same ratio

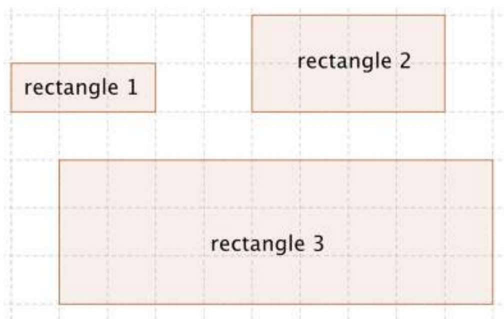
The sign ||| is used to show that two shapes are similar.

In the two quadrilateral KLMN and WXYZ, we are given KLMN ||| WZYX.

This means that $\hat{K} = \hat{W}$; $\hat{L} = \hat{Z}$; $\hat{M} = \hat{Y}$ and $\hat{N} = \hat{X}$ and $\frac{KL}{WZ} = \frac{LM}{ZY} = \frac{MN}{YX} = \frac{NK}{XW}$



Worked example:
The grid made of squares with sides 1 cm.



a) Which rectangle is similar to rectangle 1?

b) Rectangle 4 is similar to rectangle 2. It has a length of 6 cm.
What is the height of rectangle 4?

a) Compare rectangles 1 and 2. The heights are in the ratio 1 **:** 2, but the lengths are 3 **:** 4.
So they are not similar.
Rectangles 1 and 3 are similar because all the angles are the same (90°).
The heights are in the ratio 1 **:** 3 and the lengths are in the ratio 3 **:** 9 = 1 **:** 3.

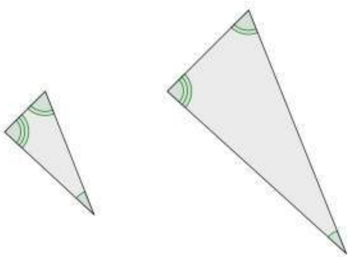
b) $\frac{\text{height of rectangle 4}}{\text{height of rectangle 2}} = \frac{\text{length of rectangle 4}}{\text{length of rectangle 2}}$ so $\frac{\text{height of rectangle 4}}{2} = \frac{6}{4}$
so length of rectangle 4 = 3 cm

Grade 7, 8 and 9

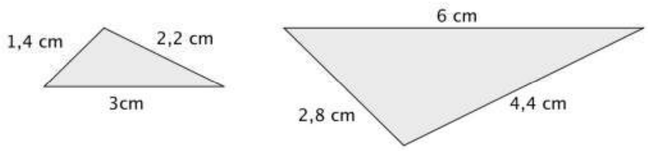
Similar triangles

Two shapes are similar if all corresponding angles are equal *and* corresponding sides are in proportion. Because of the properties of triangles, we can use some shortcuts to show two triangles are similar. There are two ways to do this:

Corresponding angles are equal



Corresponding sides are in proportion



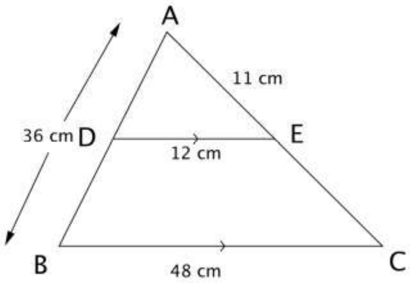
$\frac{4,4}{2,2} = \frac{6}{3} = \frac{2,8}{1,4}$

Grade 9

Worked example:

a) Show $\Delta ADE \,|||\, \Delta ABC$

$A\hat{E}D = A\hat{C}B$ (corresponding $\angle s$ DE || BC)
$\hat{A}$ is common to both triangles
$A\hat{D}E = A\hat{B}C$ (third $\angle s$ of Δs)
So $\Delta ADE \,|||\, \Delta ABC$ (all corres. $\angle s$ equal)



If two pairs of corresponding angles are equal, the third angles are equal. ($\angle s$ of a triangle add up to 180°)

b) Find the lengths of AC and AD.

$\frac{AC}{AE} = \frac{BC}{DE}$ (corres sides $\Delta ADE \,|||\, \Delta ABC$)
$\frac{AC}{11} = \frac{48}{12}$, so AC = 44 cm
$\frac{AD}{AB} = \frac{DE}{BC}$ (corres sides $\Delta ADE \,|||\, \Delta ABC$)
$\frac{AD}{36} = \frac{12}{48}$, so AD = 9 cm

c)

Grade 9

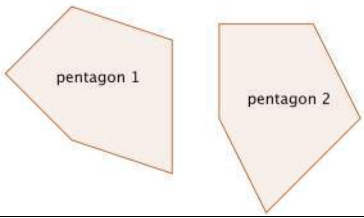
Congruent shapes

Two figures are **congruent** when they have the same shape *and* the same size.
Because they have the same shape and size, they are exactly the same so

- corresponding angles are equal, and
- corresponding sides are equal

The sign $\equiv$ is used to show that two shapes are congruent.

Example 1:
pentagon 1 $\equiv$ pentagon 2
corresponding sides and corresponding angles are equal.

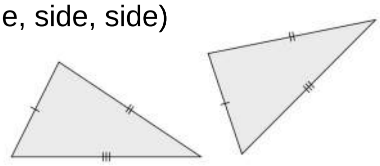


Grade 7, 8 and 9

Congruent triangles

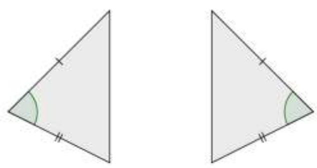
We know that two shapes are congruent if all the corresponding sides are equal *and* all the corresponding angles are equal. Because of the properties of triangles, we can use some shortcuts to show these triangles are congruent. There are four ways that we can show two triangles are congruent:

SSS (side, side, side)

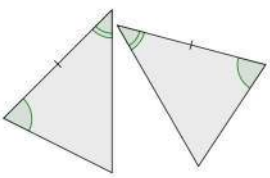


All corresponding sides are equal.

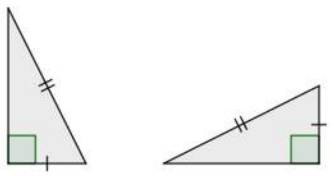
SAS (side, angle, side)
Two corresponding sides and the angle between them are equal.



AAS (angle, angle, side)
Two corresponding angles and any corresponding side are equal.



RHS (right angle, hypotenuse, side)
Both triangles have a 90° angle and equal hypotenuses and one other side equal.



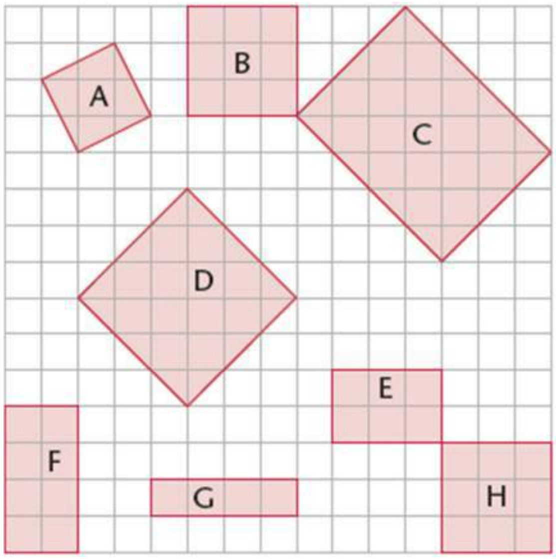
Grade 9

Questions

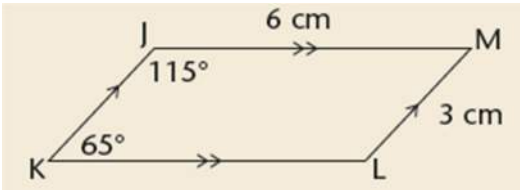
Unit 3.4: Congruence and similarity

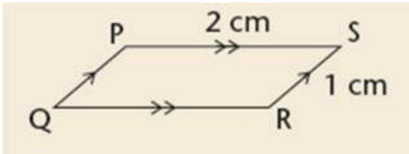
Grade 7, 8 and 9

1. Look at the shapes on the grid.
   a) Write down the letters of all the shapes that are congruent to shape B.
   b) Write down the letters of all the shapes that are similar to shape B.



2. Look at JKLM and PQRS.
   Give reasons for all answers.
   a) What type of quadrilateral is JKLM?
   b) Is JKLM ||| PQRS?
   c) What is the size of $\hat{L}$?
   d) What is the size of $\hat{S}$?
   e) What is the length of KL?





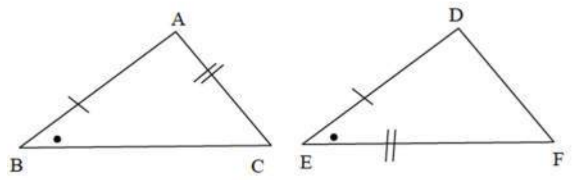
3. State whether the following are **true** or **false**.
   - If you say it is true explain why it is always true.
   - If you say it is false provide an example that shows that the statement isn't always true.

   a) If two shapes are congruent, then they are also similar.
   b) If two shapes are similar, then they are also congruent.
   c) Any two squares are congruent to each other.
   d) Any two squares are similar to each other.
   e) Any two rectangles are congruent to each other.
   f) Any two rectangles are similar to each other.
   g) All isosceles triangles are similar to each other.
   h) All isosceles triangles are congruent to each other.
   i) All equilateral triangles are similar to each other.
   j) All equilateral triangles are congruent to each other.

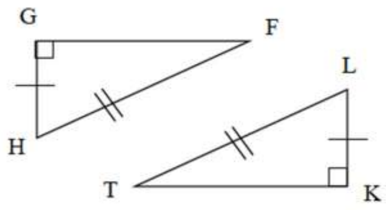
Grade 9

4. State whether each of the following pairs of triangles are similar, congruent or neither. If they are congruent, give the reason.

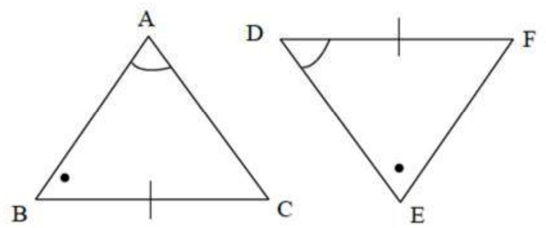
a)



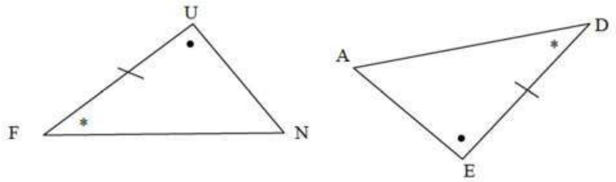
b)



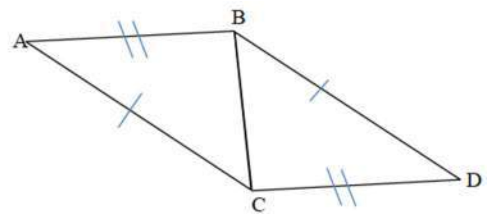
c)



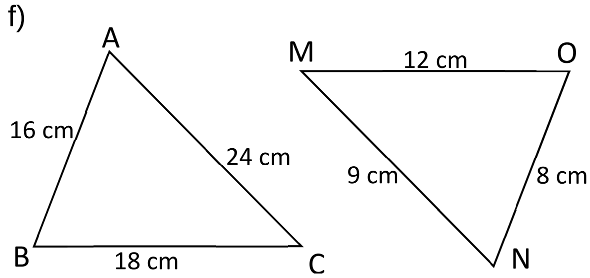
d)



e)



f)



5. State whether the following are **true** or **false**.
   - If you say it is true explain why it is always true.
   - If you say it is false provide an example that shows that the statement isn't always true.

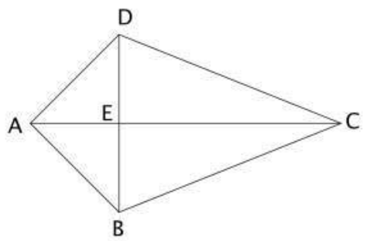
   a) Side, side, angle (SSA) is always a case for congruency in triangles.

   b) Angle, angle, angle (AAA) is a case for congruency in triangles.

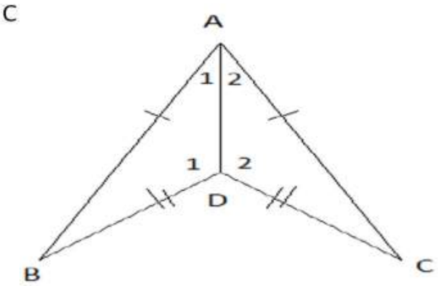
   c) If the three sides of triangle 1 are the same length as the corresponding sides of triangle 2, then the corresponding angles of triangle 1 and triangle 2 are equal.

   d) If the three angles of triangle 1 are equal to the corresponding angles of triangle 2, then the corresponding sides of triangle 1 and triangle 2 are equal.

6. If ABCD is a kite, prove that $\Delta ABC \equiv \Delta ADC$.

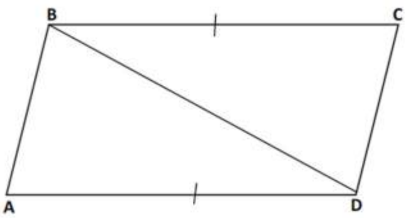


7. In the shape given, AB = AC and BD = CD. Prove that $\Delta ABD \equiv \Delta ADC$



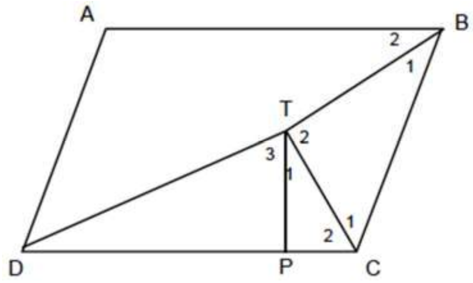
8. In the figure, $\mathrm{A\hat{B}D} = \mathrm{C\hat{D}B}$ = 90° and AD = BC.

Prove that ΔABD ≡ ΔCDB.

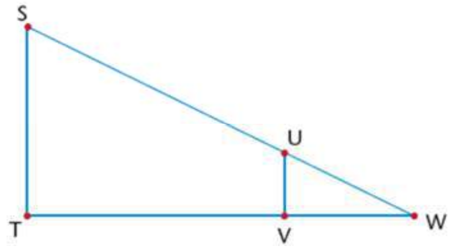


9. ABCD is a parallelogram. The bisectors of $\hat{\mathrm{B}}$ and $\hat{\mathrm{C}}$ intersect at T.
Points B, T and D do not lie on a straight line.
P is a point on DC such that $\mathrm{T\hat{P}D}$ = 90°.

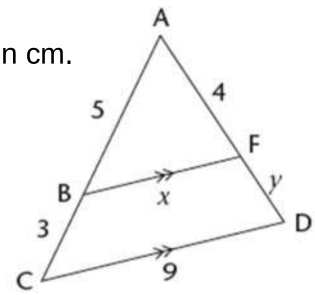
a) Prove that $\hat{\mathrm{T}}_2$ = 90°.
b) Which triangle is similar to ΔBCT?
c) If BC = 2TC and TP = 4 cm, calculate the length of BT.



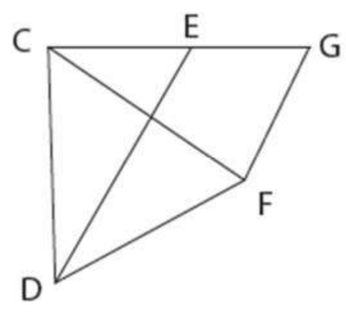
10. In the diagram, ST is a telephone pole and UV is a vertical stick. The stick is 1 m high and it casts a shadow of 1,7 m (VW). The telephone pole casts a shadow of 5,1 m (TW). Use similar triangles to calculate the height of the telephone pole.



11. Study the diagram. All numerical values represent lengths of sides in cm.
a) Explain why ΔABF ||| ΔACD
b) Use the similar triangles to find the lengths of x and y (correct to one decimal place).



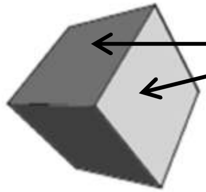
12. Given that ΔCDE ≡ ΔFCG, prove that ED || GF.
Give reasons for all statements.



Unit 3.5: 3D objects

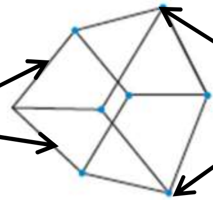
Grade 7, 8 & 9

A **polyhedron** is a three-dimensional (3D) object made of flat surfaces only. It has no curved surfaces. It has faces, edges and vertices.



A **face** is a flat surface of a 3D object.

An **edge** is a line segment where two faces of a polyhedron meet.



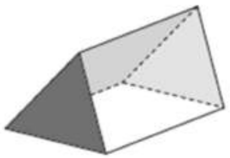
A **vertex** is the point where the edges meet.

Some special polyhedra

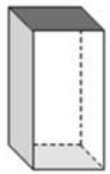
1. Prisms: The top and the base are the same polygon (e.g. a pentagon) and are parallel to each other.
Right prisms: Side faces are perpendicular to the base and top. Side faces are rectangles.

A **polygon** is a two-dimensional closed shape formed by straight lines.

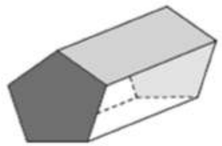
Oblique prisms: The side faces are not perpendicular to the base and top.



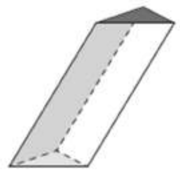
Right triangular prism
5 faces 9 edges
6 vertices



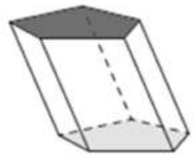
Right square-based prism
6 faces 12 edges 8 vertices



Right pentagonal prism
7 faces 15 edges
10 vertices



Oblique triangular prism
5 faces 9 edges
6 vertices



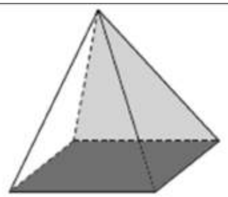
Oblique pentagonal prism
7 faces 15 edges
10 vertices

Grade 8 & 9

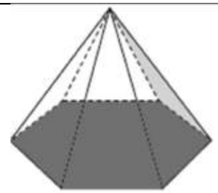
2. Pyramids: only one base; side faces are triangles and meet at a vertex called the apex.

Right pyramid: Apex lies directly above the centre of the base and all the side faces are isosceles triangles.

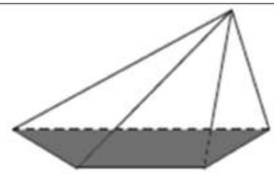
Oblique pyramid: Apex does *not* lie directly above the centre of the base; side faces are *not* necessarily isosceles triangles



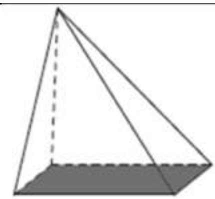
Right square-based pyramid
5 faces 8 edges 5 vertices



Right hexagon-based pyramid
7 faces 12 edges 7 vertices



Oblique quadrilateral-based pyramid
5 faces 8 edges 5 vertices



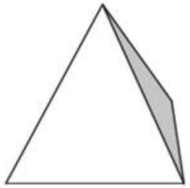
Oblique rectangle-based pyramid
5 faces 8 edges 5 vertices

3. The five Platonic solids

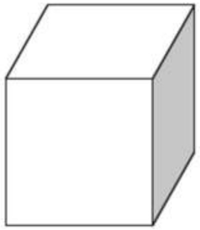
If a polyhedron has faces that are all identical regular polygons, it is called a Platonic solid.

A **regular polygon** is a polygon with equal sides and equal angles

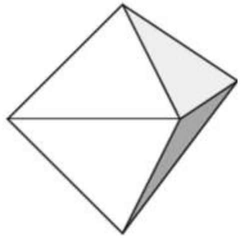
A **tetrahedron** is made of 4 equilateral triangles. It has 6 edges and 4 vertices.



A **cube** is made of 6 squares. It has 12 edges and 8 vertices.

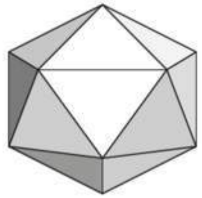


An **octahedron** is made of 8 equilateral triangles. It has 12 edges and 6 vertices.



A **dodecahedron** consists of 12 regular pentagons. It has 30 edges and 20 vertices.

An **icosahedron** consists of 20 equilateral triangles. It has 30 edges and 12 vertices.



Grade 8 & 9

4. Euler's formula:

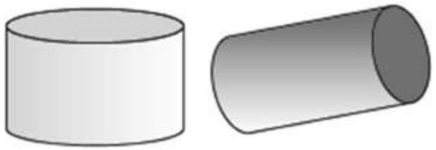
For any of the polyhedrons above, Euler's formula holds:

Euler's Formula:
(number of vertices) – (number of edges) + (number of faces) = 2

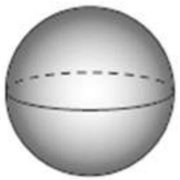
5. Some 3D objects with curved surfaces

Some solids are not polyhedrons. The solids with curved surfaces are not polyhedrons.

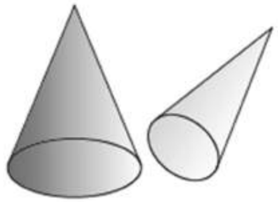
Cylinder



Sphere



Cone



6. Nets:

The net of a solid is a picture of the solid when it is 'unfolded' and laid flat. These are shown in ***Measurement Unit 4.3.***

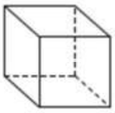
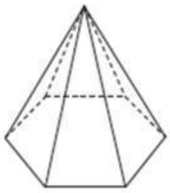
Grade 9

Questions

Unit 3.5: 3D objects and volume

Grade 7, 8 and 9

1. Name each of these 3D objects.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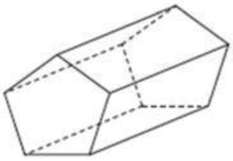
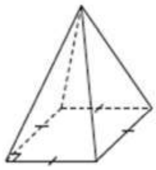
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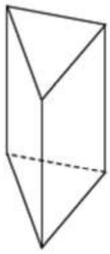
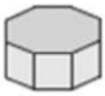
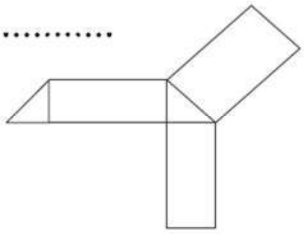
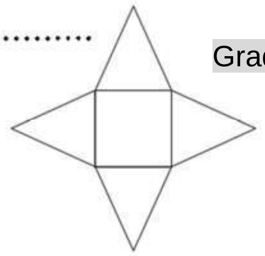
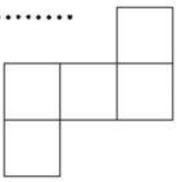
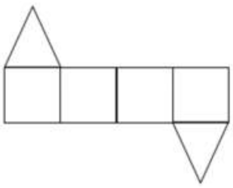


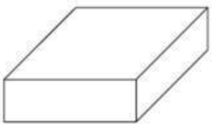
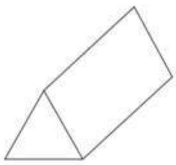
2. How many faces, edges and vertices does each of the following polyhedra have?

(a)

(b)

(c)


3. a) Can the following nets be folded to make a 3D object?
   b) If they can, give the names of the objects they make.

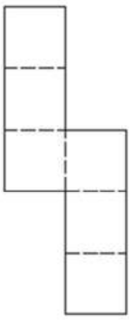
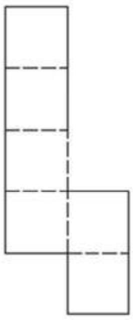
(a) ....

(b) ....

Grade 8 & 9 only
(c) ....

(d) ....


4. Draw a net for each of the following objects.
   a)

   b)


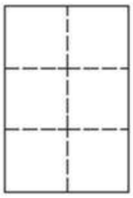
5. Which of the diagrams (a) to (h) will work as a net for a cube?

(a)

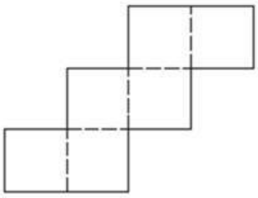
(b)

(c)

(d)


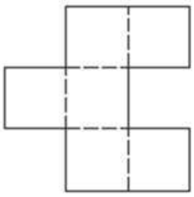
(e)



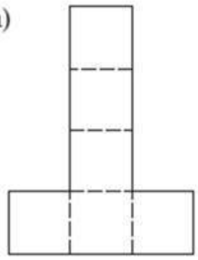
(f)



(g)



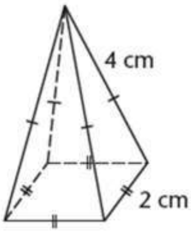
(h)



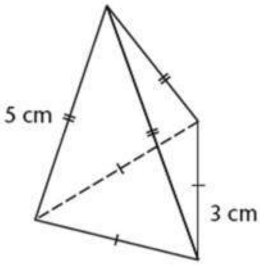
Grade 8 and 9

6. Draw a net for each of the following objects.

   a) square based pyramid



   b) triangular pyramid



7. Which of the following objects could be Platonic solids?

A.

B.

C.

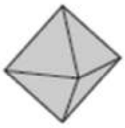
D.

E.

F.

G.

H.



I.

J.

Grade 9

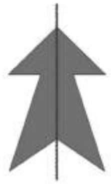
8. Grade 9 learners were asked to represent a 3D object and give the class clues about their polyhedron. Use the clues to name each of their objects:

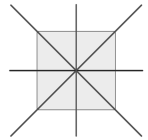
   a) I have 6 faces and they are all the same size.
   b) I have 6 faces and 12 edges. I am not a cube.
   c) I have 8 edges and I have 5 vertices.
   d) I have 6 edges and 4 vertices.
   e) I have 8 faces and I am a Platonic solid.
   f) I do not have any edges.
   g) My faces are made only of regular pentagons.

Unit 3.6: Transformation Geometry

Symmetry

- A line of symmetry divides a figure into two identical halves. The two halves are mirror images of each other.
- If you fold the figure along the line of symmetry, the two parts will fit exactly on top of each other.
- A shape can have more than one line of symmetry.

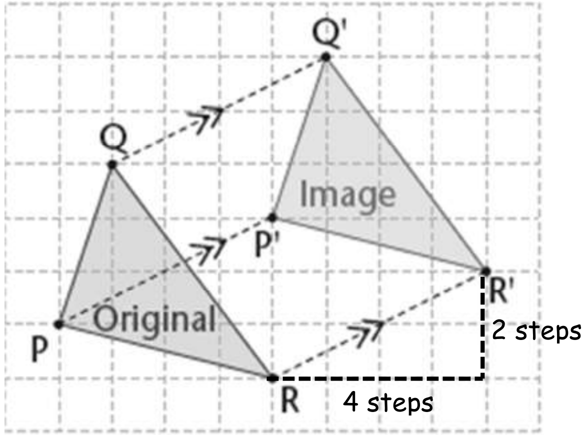




Moving figures

In transformation geometry, we move or 'transform' shapes. The *'moved'* shape in the new position is called the *image* of the original shape. We move shapes without changing their size or shape in three ways: translation, reflection or rotation.

Moving by translation (shifting or sliding)

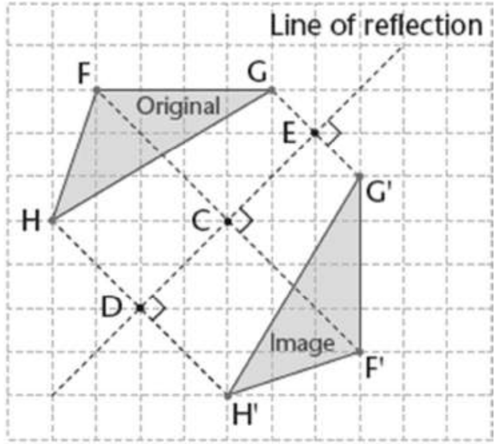


Triangle PQR is translated (slide) to produce the image P'Q'R'. It is translated along a straight line. So the lines $PP' = QQ' = RR'$ and $PP' \parallel QQ' \parallel RR'$

We can describe a translation by saying how many units up (or down) and right (or left) the object has moved. Each point of triangle PQR has moved 4 steps to the right and two steps up.

Moving by reflection

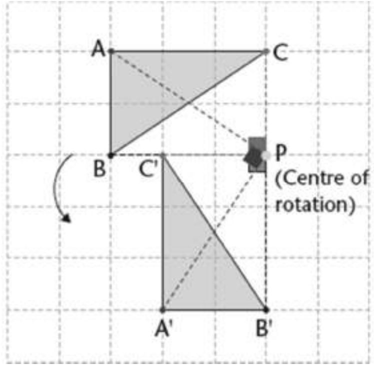
(flipping or turning over)



If you fold along the line of reflection, the image ΔF'G'H' will fit exactly on the original ΔFGH .

The perpendicular distance from the line of reflection to a point is equal to the perpendicular distance from the line of reflection to the image of the point i.e. $CF = CF'$ and $DH = DH'$ and $EG = EG'$

Moving by rotation (turning)

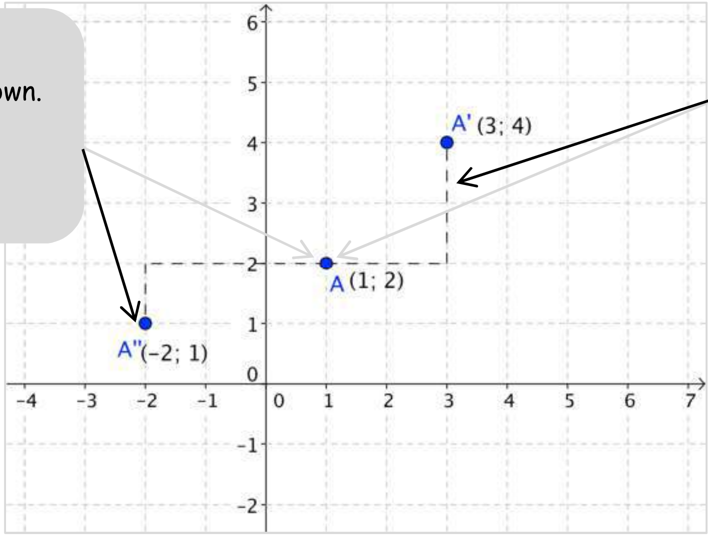


ΔABC is rotated around the centre of rotation P. Each point of ΔABC is rotated about the point P through 90° anti-clockwise. $A\hat{P}A'$, the angle of rotation, is 90°.

Grade 7, 8 & 9

Translation

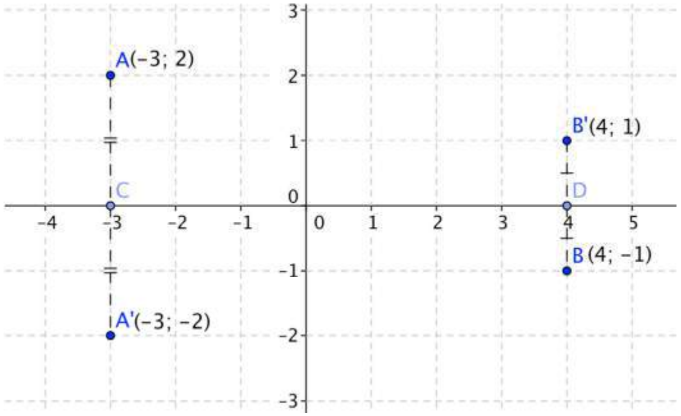
To translate (shift/slide) A to A'':
Move A 3 steps to the left, 1 step down.
Find the coordinates of A'':
A (1; 2) ⟶ 3 units left, 1 unit down
⟶ A' (1 – 3; 2 – 1) = A' (-2; 1)



To translate (shift/slide) A to A':
Move 2 steps to the right, 2 steps up.
Find the coordinates of A' :
A (1; 2) ⟶ 2 units right, 2 units up
⟶ A' (1 + 2; 2 + 2) = A' (3; 4)

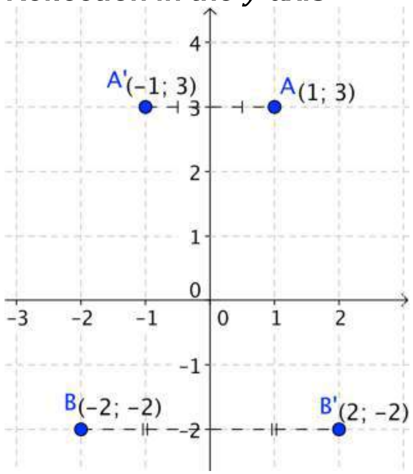
A rule for translating any point $(x; y)$:
If the point moves 5 units up and 3 units left, the image is $(x - 3; y + 5)$

Reflection in the x-axis



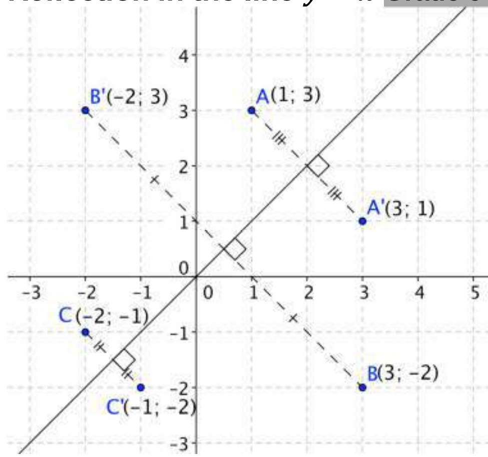
We see that if we reflect any point $(x; y)$ in the x-axis its image is $(x;\ -y)$

Reflection in the y-axis



We see that if we reflect any point $(x; y)$ in the y-axis its image is $(-x;\ y)$

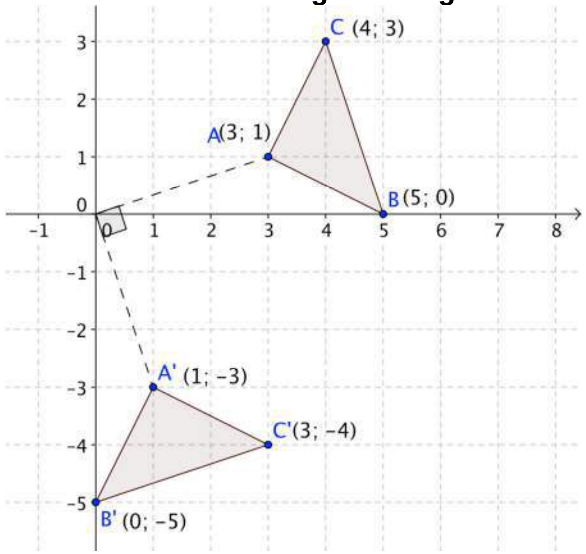
Reflection in the line $y = x$ Grade 9 only



We see that if we reflect any point $(x; y)$ in the line $y = x$ its image is $(y;\ x)$

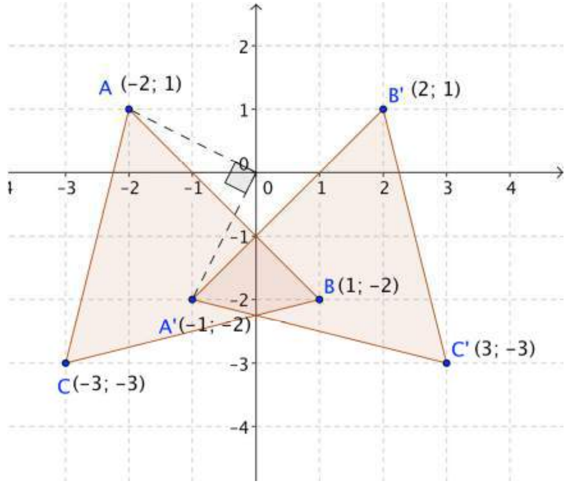
Grade 8 & 9

Rotation about the origin through 90° clockwise



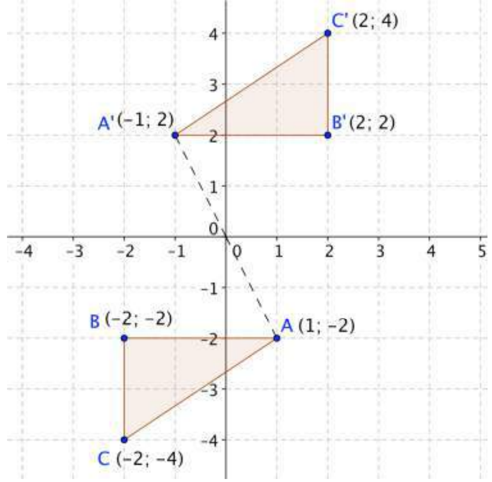
When a point $(x;\ y)$ is rotated about the origin through 90° clockwise, its image is $(y; -x)$

Rotation about the origin through 90° anti-clockwise



When a point $(x;\ y)$ is rotated about the origin through 90° anti-clockwise, its image is $(-y;\ x)$

Rotation about the origin through 180° (anticlockwise or clockwise)

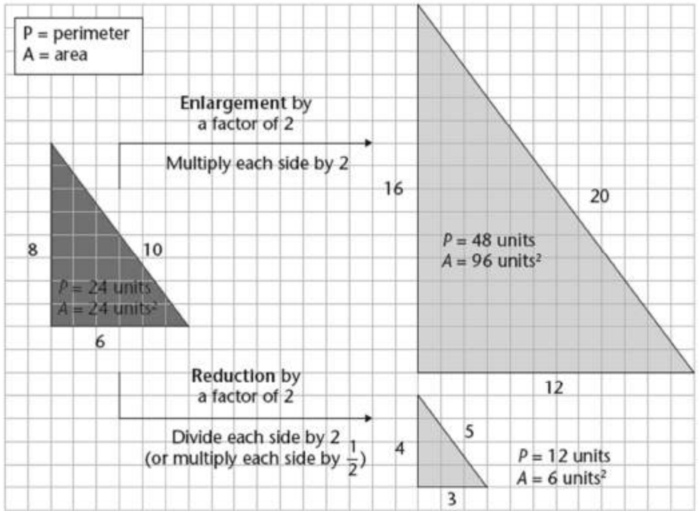


We can see that when a point $(x;\ y)$ is rotated about the origin through 180° its image is $(-x;\ -y)$

Grade 8 & 9

Enlargement and reduction: Transformations that change the size but not the shape of the figure.

- An enlargement or a reduction of a shape has all sides **in proportion** to the corresponding sides of the original shape.
- The lengths of all sides of the original shape can be multiplied by the same number to produce the image. This is called the **scale factor**.
- To work out the scale factor, use $\frac{\text{side length of image}}{\text{corresponding side length of original}}$
  - If the scale factor is > 1, the image is an enlargement
  - If the scale factor is < 1, the image is a reduction
- The original shape and the enlarged or reduced shape are **similar** (their corresponding angles are all equal)
- Perimeter of image = perimeter of original × scale factor
- Area of image = area of original × (scale factor)^2



Grade 7, 8 & 9

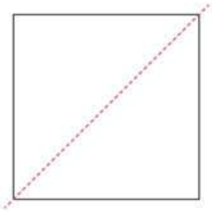
Questions

Unit 3.6: Transformation geometry

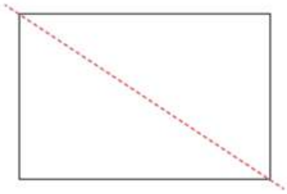
Grade 7 (8 and 9 revision)

1. In each diagram, is the dotted line a line of symmetry? If it is not, draw a line of symmetry if this is possible.

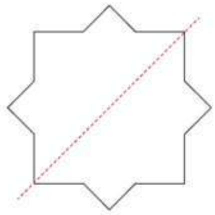
a)



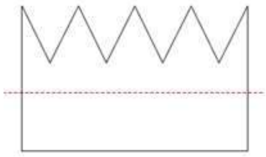
b)



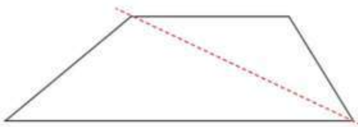
c)



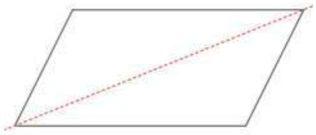
d)



e)

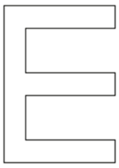


f)

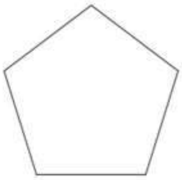


2. Draw all the lines of symmetry for each shape.

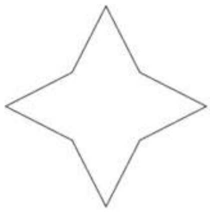
a)



b)

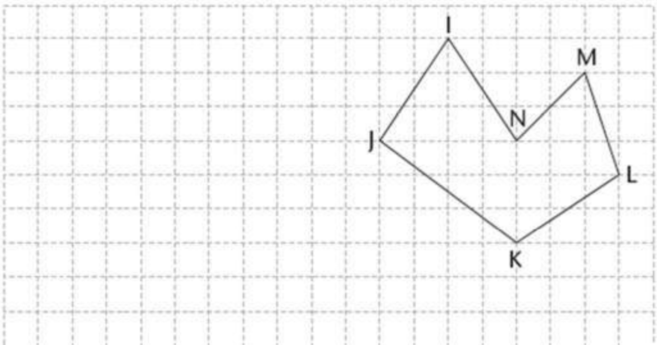


c)

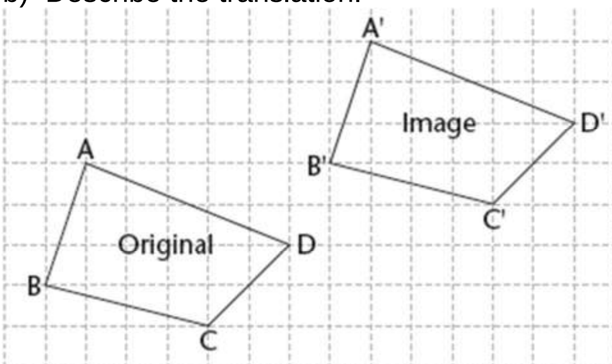


Grade 7, 8 and 9

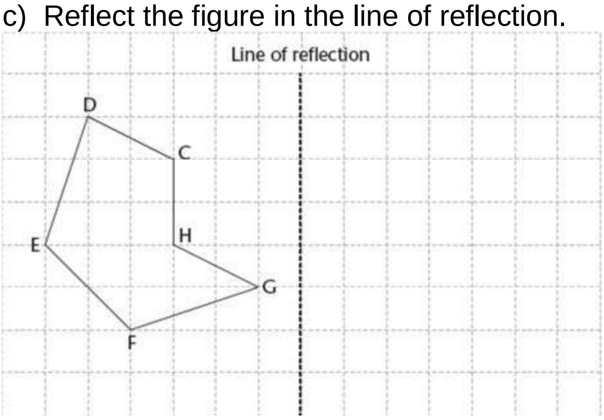
3a) Translate the figure 8 units to the left and 2 units down.



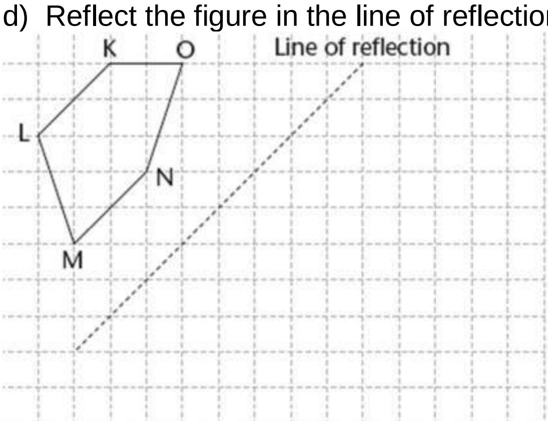
b) Describe the translation.



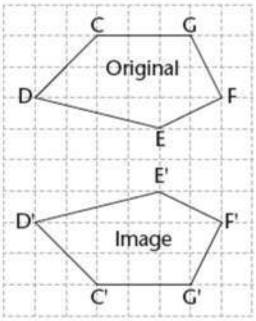
c) Reflect the figure in the line of reflection.



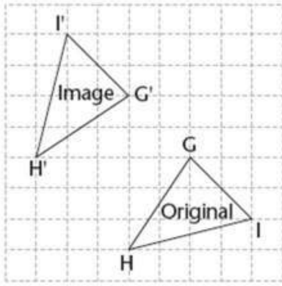
d) Reflect the figure in the line of reflection.



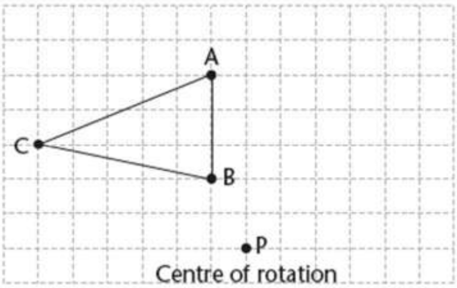
4a) Draw the line of reflection.



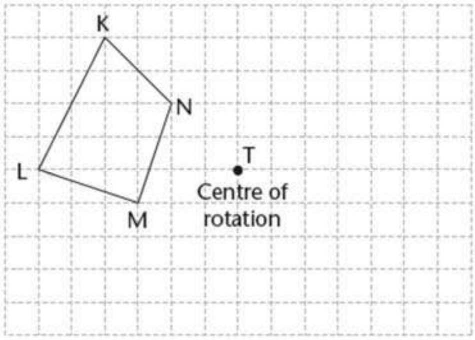
b) Draw the line of reflection.



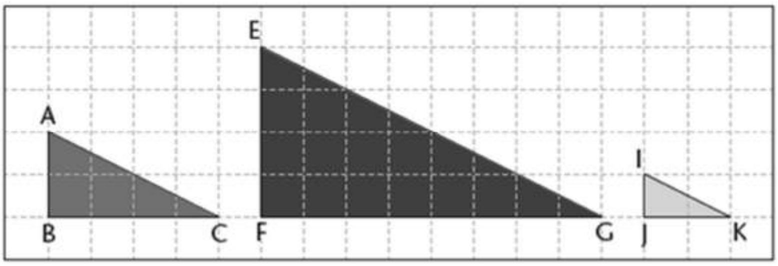
5a) Rotate the triangle 90^o clockwise about the point P.



b) Rotate the quadrilateral 180^o about point T.



6. Look at the triangles and answer the questions.



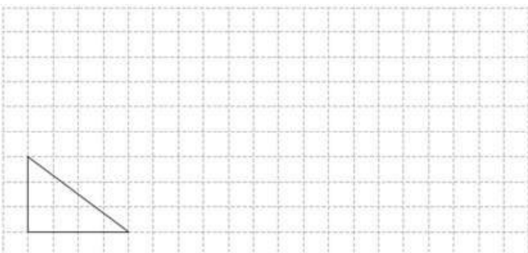
a) How many times longer is FG than BC?
How many times longer is EF than AB?
How many times longer is EG than AC?

b) How many times shorter is JK than BC?
How many times shorter is IJ than AB?
How many times shorter is IK than AC?

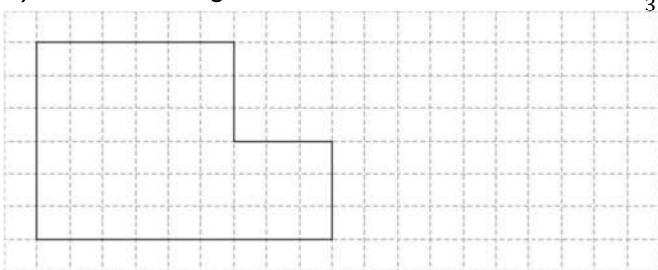
c) Is ΔEFG an enlargement of ΔABC? Explain your answer.

d) Is ΔIJK a reduction of ΔABC? Explain your answer.

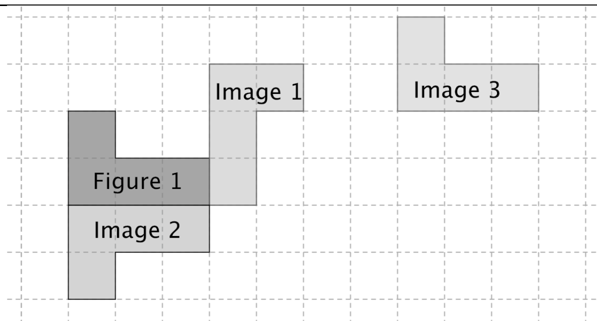
7a) Enlarge the triangle below with a scale factor of 2.



b) Resize the figure below. Use a scale factor of $\frac{1}{3}$



8. Figure 1 has been transformed in three different ways to form images 1, 2 and 3. Describe in words the transformation that has produced each of these images.

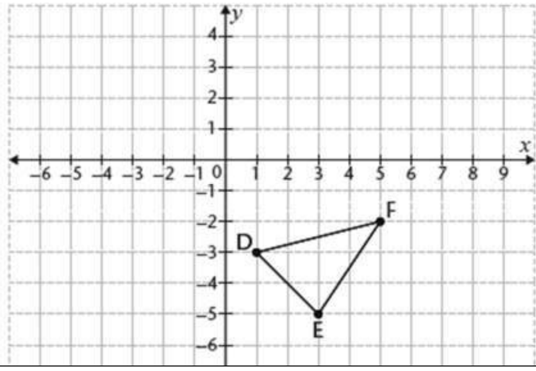


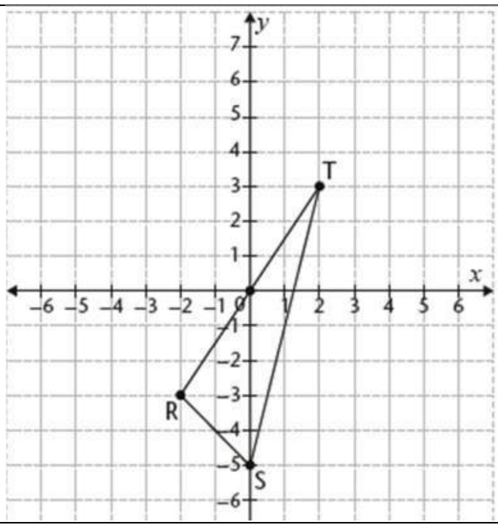
Grade 8 and 9

9a) Translate ΔDEF 4 units to the left and 2 units down. What are the coordinates of the vertices of ΔD'E'F'?

b) Reflect ΔDEF over the x-axis. What are the coordinates of ΔD"E"F"?

c) Reflect ΔDEF over the y-axis. What are the coordinates of ΔD'''E'''F'''?





10a) Rotate the triangle 180° about the origin. Write down the coordinates of the vertices.

b) Rotate the triangle 90° anticlockwise about the origin. Write down the coordinates of the vertices.

c) Resize the triangle using the origin as the centre of resizing and a scale factor of $\frac{1}{2}$. Write down the coordinates of the vertices.

11. ΔABC has vertices A(–1; –5), B(–3; 2) and C (2; 1). If Δ ABC is transformed to create the image ΔA'B'C', describe the transformation in words, where the vertices of ΔA'B'C' are:

	a)	b)	c)	d)	e)	f)	g)
A'	(0; –6)	(1; 5)	(–3; –15)	(1; –5)	(1; –5)	(5; –1)	(–5; –1)
B'	(–2; 1)	(3; –2)	(–9; 6)	(3; 2)	(–1; 2)	(–2; –3)	(2; –3)
C'	(3; 0)	(–2; –1)	(6; 3)	(–2; 1)	(4; 1)	(–1; 2)	(1; 2)

Note: It helps to draw the triangles on a grid.

12. A rectangle with width 4 cm and length of 6 cm is enlarged to make a rectangle with a width of 10 cm and length of 15 cm.
   a) What scale factor has the rectangle been enlarged by?
   b) How many times bigger is the area of the enlarged rectangle than that of the original rectangle?

Grade 9

13. Give the coordinates of the images of points A, B and C after the transformation.

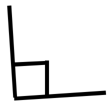
	Translation 4 units up and 3 units to the left	Rotated about the origin through 90° clockwise	Reflected in the line $y = x$
A (–2; 1)			
B (3; 4)			
C (5; –2)			

14. ΔABC has vertices A(–1; –5), B(–3; 2) and C (2; 1). ΔABC is reflected in the line $y = x$, then this new triangle is translated 1 unit down and 3 units to the right; then it is enlarged by a factor of 5 to make ΔA'B'C' . What are the coordinates of the vertices of ΔA'B'C'?

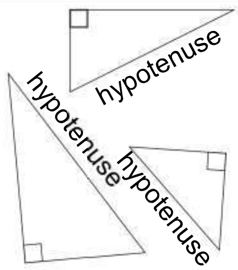
Unit 4.1: Pythagoras theorem

→ A **right-angled triangle** has one angle equal to 90°.

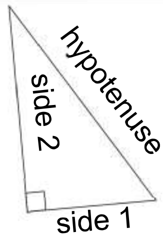
→ We show a 90° angle with a small square.



→ The side opposite the 90° angle is called the **hypotenuse.**



The theorem of Pythagoras:
In a right-angled triangle, a square formed on the hypotenuse has the same area as the sum of the area of the two squares formed on the other sides of the triangle.



In a right-angled triangle
$(\text{hypotenuse})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$

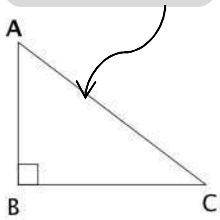
The converse of the theorem of Pythagoras:
The theorem of Pythagoras also works the other way.
If you find the sides of a triangle have the 'squared' relationship, then you can say that the triangle is right-angled.

If $(\text{longest side})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$ then the triangle is **right-angled**.

Worked examples:

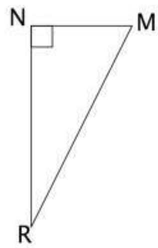
1. If AB = 3 cm and BC = 4 cm, calculate the length of AC.

AC is the hypotenuse.



$AC^2 = AB^2 + BC^2$ (Pythag)
$AC^2 = 3^2 + 4^2$
$AC^2 = 9 + 16 = 25$
So $AC = \sqrt{25} = 5$ cm

2. If MN = 12,4 mm and MR = 28,3 mm, calculate the length of NR correct to one decimal place.



$MN^2 + NR^2 = MR^2$ (Pythag)
$(12{,}4)^2 + NR^2 = (28{,}3)^2$
$NR^2 = (28{,}3)^2 - (12{,}4)^2$
$= 800{,}89 - 153{,}76 = 647{,}13$
So $NR = \sqrt{647{,}13} = 25{,}4387$ ...
NR = 25,4 mm (rounded to 1 decimal place)

3. For each triangle, work out if it is a right-angled triangle:

a) A triangle with side lengths 7, 9 and 10:
$(\text{longest side})^2 = 10^2 = 100$
$(\text{side } 1)^2 + (\text{side } 2)^2 = 7^2 + 9^2$
$= 49 + 81 = 130$
$100 \neq 130$
So it cannot be a right-angled triangle.

b) A triangle with side lengths 3, 4 and 5:
$5^2 = 25$ and $3^2 + 4^2 = 9 + 16 = 25$
So the triangle is right-angled

c) A triangle with side lengths 5, 12 and 13:
$13^2 = 169$ and $5^2 + 12^2 = 25 + 144 = 169$
So the triangle will be right-angled

Sets of whole numbers that are sides of right angle triangles (like 3; 4 and 5 or 5; 12 and 13) are called Pythagorean triples.

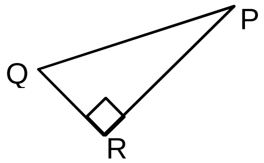
Grade 8 and 9

Questions

Unit 4.1: Pythagoras' theorem for right angled triangles

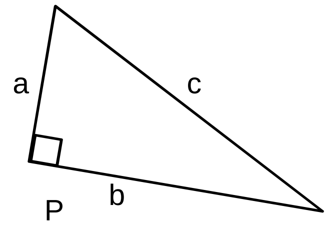
Grade 8 and 9

1. Complete the sentences:
   a) Pythagoras' theorem only works in ____________________ triangles.
   b) The side opposite the right angle in a right-angled triangle is called the _________________.
   c) The longest side in a right-angled triangle is called the _________________.
   d) Which side of this right-angled triangle is the hypotenuse? ____________

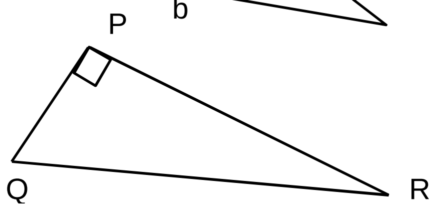


   e) Can you have 2 right angles in a triangle? Explain your answer.
   f) Complete this Pythagorean triple: 3; 4; _____
   g) Complete this Pythagorean triple: 5; 12; _____
   h) Complete this Pythagorean triple: _____; 8; 10

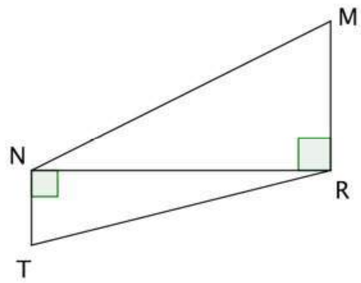
2. Complete these statements for the triangle shown here:
   a) $a^2 + b^2 =$ ___________
   b) $c^2 - a^2 =$ ___________
   c) $a^2 =$ __________________
   d) $c^2 =$ __________________



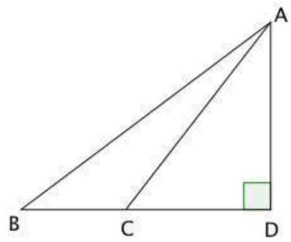
3. Δ PQR has QR = 13 mm; PQ = 5 mm. Calculate PR.



4. If MN = 102 km; MR = 62 km and NT = 48 km, determine the distance from T to R.



5. In this diagram $\widehat{D} = 90°$, AB = 25 cm; AD = 8 cm and AC = 8,54 cm.
   Calculate the length of BC (correct to the nearest whole number.)



6. A rectangle has a width of 25 cm and a length of 45 cm. What is the length of the rectangle's diagonal?

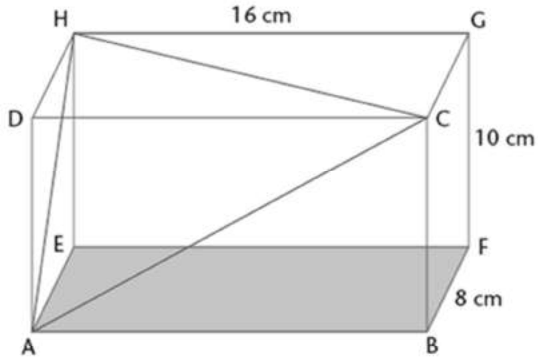
7. Is it possible to draw a *square* with sides that are 3 cm in length and with a diagonal that is 5 cm in length? Explain why you say so.

8. A rectangular prism is made of glass. It has a length of 16 cm, a height of 10 cm and a breadth of 8 cm. ABCD and EFGH are two of its faces. ∆ACH is drawn inside the prism.

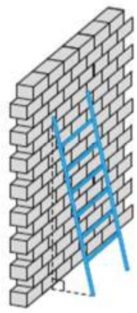
Is ∆ACH right-angled? Answer the questions to find out.

a) Calculate the lengths of the sides of ∆ACH.
<u>Hint</u>: All three sides of the triangle are diagonals of rectangles: AC is in rectangle ABCD, AH is in ADHE and HC is in JDCG.

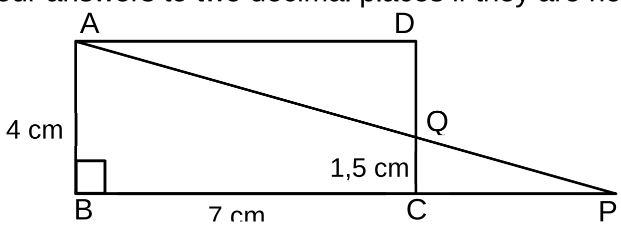
b) Is ∆ACH right-angled? Explain your answer.



9. A ladder of length 5 m is placed at an angle against a wall.
a) The bottom of the ladder is 1 m away from the wall. How far up the wall will the ladder reach? Round off to two decimal places.
b) If the ladder reaches a height of 4,5 m against the wall, how far away from the wall is it placed? Round off to two decimal places.



10. ABCD is a rectangle with AB = 4 cm, BC = 7 cm and CQ = 1,5 cm.
Round off your answers to two decimal places if they are not whole numbers.



a) What is the length of QD?
b) If CP = 4,2 cm, calculate the length of PQ.

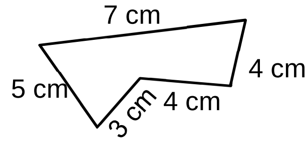
Unit 4.2: Perimeter and Area

Perimeter:

The perimeter is total **distance around** any closed 2-D shape.
The perimeter is measured in metres (m), millimetres (mm), centimetres (cm) or kilometres (km).

Converting units
1 km = 1 000 m
1 m = 100 cm
1 cm = 10 mm

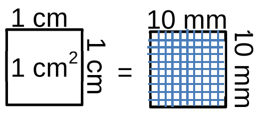
Example:



Perimeter of shape
= 5 + 7 + 4 + 4 + 3 = 23 cm

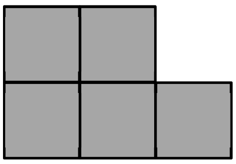
Area: Area is the size of the surface of a flat 2-D shape. So area is the number of **square units** that fit **onto** a shape.
Area is measured in square metres (m^2), square millimetres (mm^2), square centimetres (cm^2) or square kilometres (km^2).

Converting square units

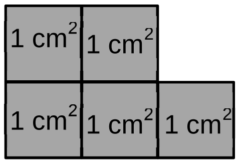


$1\ cm^2 = 10\ mm \times 10\ mm = 100\ mm^2$
$1\ m^2 = 100\ cm \times 100\ cm$
$= 10\ 000\ cm^2$
$1\ km^2 = 1\ 000 \times 1\ 000 = 1\ 000\ 000\ m^2$

Worked example:
Find the area and perimeter of the shape. Each small square has sides of 1 cm.

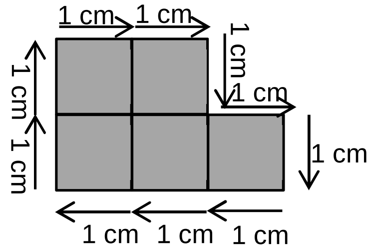


Area



Area of shape = 5 cm^2

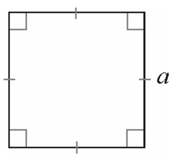
Perimeter



Perimeter of shape = 10 cm

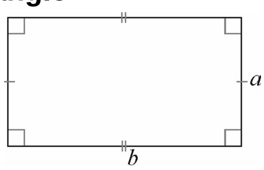
Formulas to calculate perimeter and area

Square



Perimeter = 4 × length = $4a$
Area = length × length = a^2

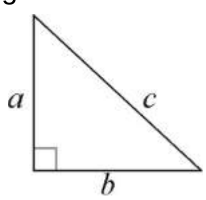
Rectangle



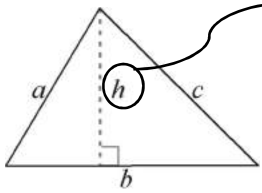
Perimeter = $2a + 2b$
Area = length × breadth = ab

Right-angled triangle

Perimeter = $a + b + c$
Area = $\frac{1}{2} \times$ base $\times \perp$ height
$= \frac{1}{2} \times b \times a$



Triangle



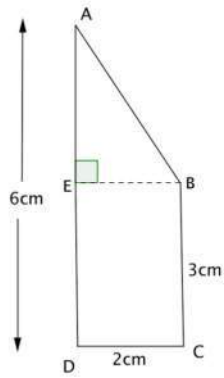
h is the perpendicular height from any vertex to the side opposite.

Perimeter = $a + b + c$
A = $\frac{1}{2} \times$ base $\times \perp$ height $= \frac{1}{2} \times b \times h$

Grade 7, 8 and 9

Grade 7, 8 & 9

Worked example:
Calculate:
a) the total area of this shape (Grade 7, 8 & 9)
b) the total perimeter of this shape. (Grade 8 & 9)



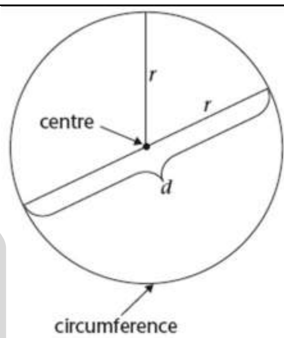
a) Area of rectangle = $2 \times 3 = 6$ cm^2
EB = 2 cm (opposite sides of rectangle are equal)
AE = AD – ED = 6 cm – 3 cm = 3 cm
So area of triangle = $\frac{1}{2} \times b \times h$ = ½ (2×3) = 3 cm^2
Total area of shape = 6 cm^2 + 3 cm^2 = 9 cm^2

b) AB^2 = $3^2 + 2^2 = 13$ (Pythagoras) (Grade 8 & 9)
So AB = $\sqrt{13}$ cm
Perimeter = 3 cm + 2 cm + 6 cm + $\sqrt{13}$ cm
= 14,6 cm

Grade 8 & 9

The circle

r is the radius of the circle
d is the diameter of the circle and $d = 2r$
π = 3,141592… It is an irrational number.
Your calculator gives an approximate value of π.

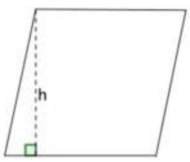


The perimeter of a circle is called a **circumference**.

Circumference = $2\pi r$
= πd
Area (A) = πr^2

Grade 9

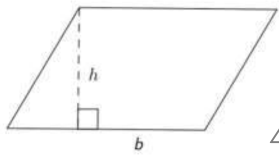
Rhombus

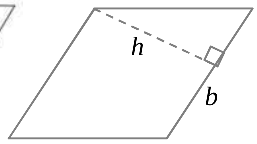


A = $b \times h$

h is the perpendicular height from any vertex to the side opposite.

Parallelogram



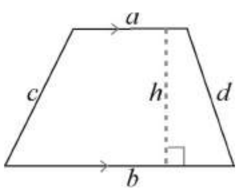


A = $b \times h$

Area of rhombus or parallelogram = $b \times h$

Trapezium

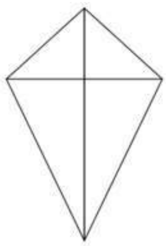
A = $\frac{1}{2} \times (a + b) \times h$
= $\frac{1}{2} h (a + b)$



$(a + b)$ is sum of two parallel sides.

Kite

A = ½ diagonal$_1$ × ½ diagonal$_2$



Questions

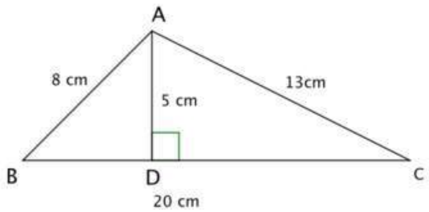
Unit 4.2 Perimeter and Area

Grade 7, 8 and 9

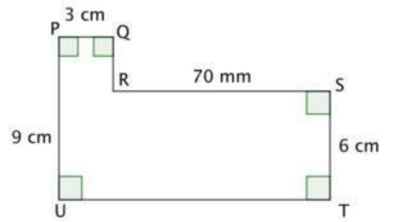
1. Convert:
   a. 900 mm to ________ cm
   b. 7 km to ________metres
   c. 7,6 m to ________ cm
   d. 1 km 320 m to _________ cm
   e. 943 217 m to __________ km

2. Convert:
   a. 5 cm^2 to _________ mm^2
   b. 8,25 m^2 to _____________cm^2
   c. 245 000 000 m^2 to __________km^2
   d. 5,32 km^2 to _________________ m^2
   e. 347 012 mm^2 to _____________ cm^2

3. What is the area of a rectangle that has a length of 4,7 m and a breadth of 2,3 m?
4. Calculate the perimeter of a rectangle that has a breadth of 15,3 cm and a length of 19,2 cm.
5. If the area of a rectangle is 13,23 cm^2 and the length is 6,3 cm, calculate the breadth.
6. If the Area of a triangle is 17,5 m^2 and the base is 5 m, calculate the length of the perpendicular height.



7. In the diagram BC = 20 cm; AC = 13 cm; AB = 8 cm and AD = 5cm.
   a) Calculate the perimeter of ΔABC
   b) Calculate the area of ΔABC



8. a) Determine the perimeter of the shape PQRSTU
   b) Determine the area of the shape PQRSTU

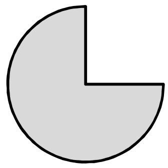
Grade 8 and 9

9. a) If $\hat{B} = 90°$, AB = 27,9 mm and BC = 14,8 mm, determine the area of Δ ABC in cm^2 .
   b) If $\hat{B} = 90°$, AB = 27,9 mm and BC = 14,8 mm, determine the perimeter of Δ ABC in cm.

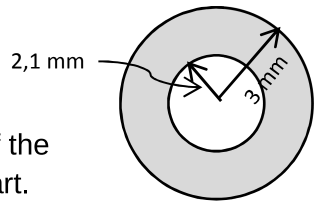
10. Refer to triangle ABDC (Question 7). In the diagram BC = 20 cm; AC = 13 cm; AB = 8 cm and AD = 5cm.
    a) Calculate the perimeter of ΔABD b) Calculate the area of ΔABD

Note: *For questions 11 – 14, use π on your calculator and round off to 2 decimal places*

11. Calculate a) the area and b) the circumference of a circle that has a radius of 7 cm.
12. Calculate a) the area and b) the circumference of a circle that has a diameter of 36 m.
13. If the area of a circle is 50,27 cm^2, calculate (correct to 2 decimal places)
    a) the length of the radius b) the diameter c) the circumference



14. The diameter of the circle is 16 m.
    a) A quarter of the circle is removed. What fraction of the circle is left?
    b) Calculate the area of this shape.
    c) Calculate the perimeter of this shape.

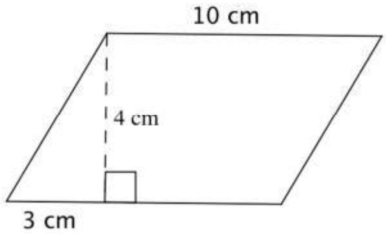


15. The radius of the smaller circle is 2,1 mm and the radius of the larger circle is 3 mm. Determine the area of the shaded part.

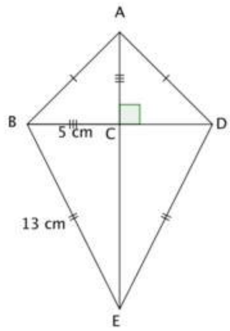
Grade 9

16. Calculate the area and perimeter of each of the shapes below:

a) parallelogram

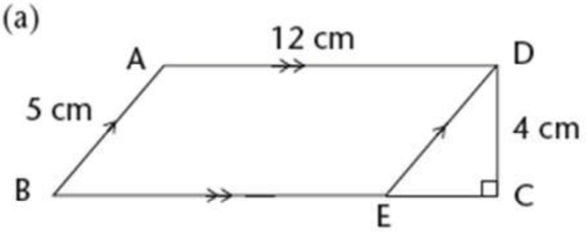


b) Kite with AC = BC; BC = 5cm and BE = 13 cm

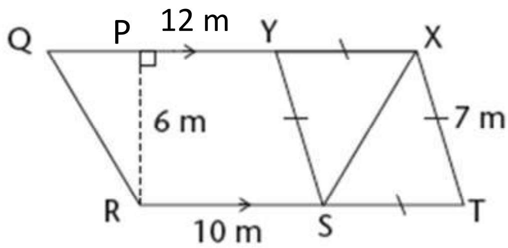


17. The perimeter of a rectangle is 30 m and its area is 36 m^2. If the length and breadth are both doubled:
    a) determine the perimeter of the new, enlarged rectangle.
    b) determine the area of the new, enlarged rectangle.
18. If the radius of a circle is doubled,
    a) how does this change the circumference of the circle?
    b) how does this change the area of the circle?
19. Calculate the area and perimeter of each of the following shapes. Round off your answers to two decimal places where necessary.

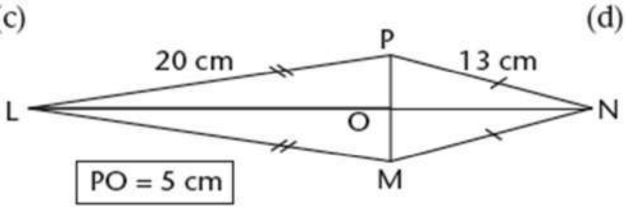
(a)



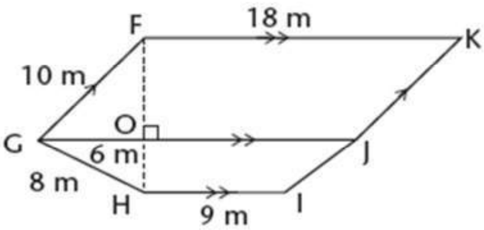
(b)



(c)



(d)



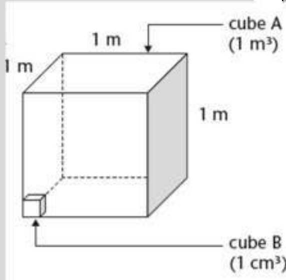
Unit 4.3: 3D shapes and volume

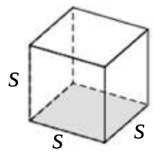
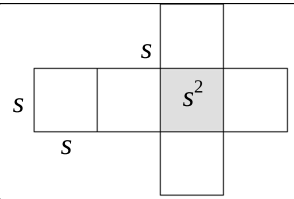
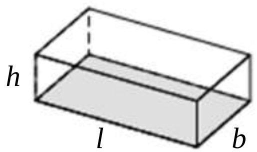
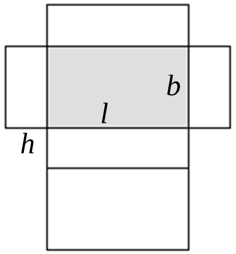
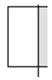
- Volume measures the *amount of space* occupied by a 3D object.
- We use cubes (or cubic units) as the unit to measure volume.
  A cube with edges of 1 cm has a volume of
  1 cubic centimetre (cm^3)

Units of measurement

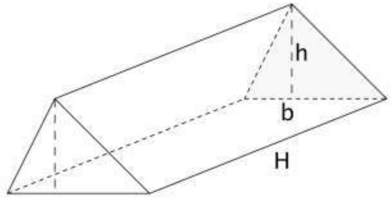
1 m^3 = 1 000 000 cm^3 1 cm^3 = 0,000001 m^3
1 cm^3 = 1 000 mm^3 1 mm^3 = 0,001 cm^3
1 litre (l) = 1 000 cm^3
1 kilolitre (kl) = 1 000 l = 1 000 000 cm^3

Why is 1 m^3 = 1 000 000cm^3?
The big cube (A) has edge lengths of 1 m and the small cube (B) has edge lengths of 1 cm. We can fit 100 small cubes along the length of the big cube, and along the breadth and along the height.
So total 1 cm^3 cubes in 1 m^3
= 100 x 100 x 100 = 1 000 000

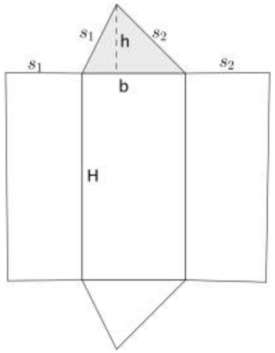


3D object: right prisms	Net	Surface area	Volume
A right prism is a geometric solid that has a polygon as its base and vertical sides perpendicular to the base. The base and top surface are the same shape and size.	The net of a solid is a picture of the solid when it is "unfolded" and laid flat.	The surface area of a solid is the sum of the areas of all its faces (or outer surfaces). So we can add up the area of all the shapes in the net of the solid	Volume measures the amount of space an object takes up. Volume of a right prism = area of base $\times$ height
Cube has a square base s = length of sides 		Surface area = 6 $\times$ area of square = $6s^2$	Volume = area of square $\times$ height = $s^2 \times s$ = s^3
Rectangular prism has a rectangle as base  l = length of rectangle b = breadth of rectangle h = height of prism		Surface area = 2  + 2  + 2  $= 2 \times l \times b + 2 \times b \times h + 2 \times l \times h$ $=2lb + 2bh + 2lh$	Volume = area of rectangle $\times$ height $= l \times b \times h$ $= lbh$

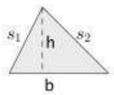
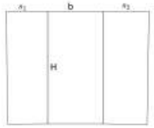
Grade 7, 8 & 9



Triangular prism has a triangle as base
h = ⊥ height of triangle,
H = height of prism, b = base of triangle



Surface area

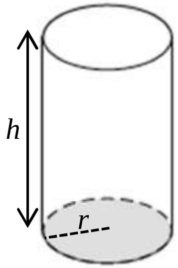
= 2  + 

= 2 $(\frac{1}{2}\text{bh})$ + (perimeter of triangle) × H
= $\text{bh} + (\text{s}_1 + \text{s}_2 + \text{b}) \times \text{H}$

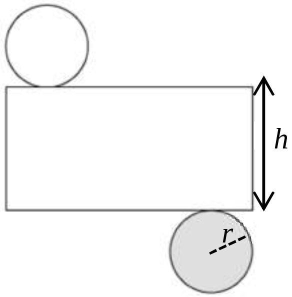
Volume
= area of triangle × height
= $\frac{1}{2} \times b \times h \times H$
= $\frac{1}{2}bhH$

Grade 8 & 9

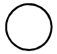
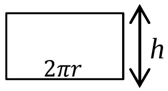
Cylinder has a circle as base



r = radius of the circle
h = height of the cylinder



Surface area

= 2  + 

= $2\pi r^2 + 2\pi rh$

Volume = area of circle x height
= $\pi r^2 h$

Remember:
Area of  = πr^2
Circumference = 2 πr

The length of the rectangle is the same as the circumference of the circle

Grade 9

Worked examples:

1. A carton that is the shape of a rectangular prism is 30 cm long, 20 cm wide and 10 cm high.
   a) Calculate the volume of the carton
   b) Calculate the surface area of the carton.

2. A cylinder has a radius of 3 cm and a height of 10 cm.
   a) Calculate the volume of the cylinder.
   b) What happens to the volume of the cylinder if the radius and height are doubled?

Solutions

1a) Volume = $l \times b \times h = 30\ \text{cm} \times 20\ \text{cm} \times 10\ \text{cm} = 6\,000\ \text{cm}^3$
b) Surface area = $2lb + 2bh + 2lh$
$= 2(30)(20) + 2(20)(10) + 2(30)(10) = 2\,200\ \text{cm}^2$

Grade 7, 8 & 9

a) Volume of cylinder = $\pi r^2 h = \pi \times 3^2 \times 10 = 90\pi \approx 282{,}74 \text{cm}^3$

b) radius = 6 cm and height = 20 cm
Volume = $\pi r^2 h = \pi \times 6^2 \times 20 = 720\pi \approx 2\,261{,}94\ \text{cm}^3$.
2 261,94 ÷ 282,74 ≈ 8
So new volume is 8 times bigger than volume of the original cylinder.

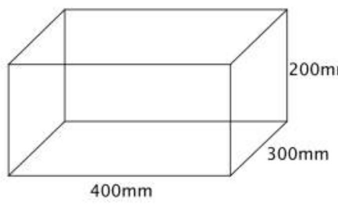
Grade 9

Questions

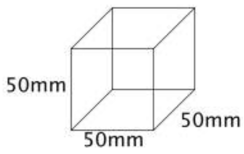
Unit 4.3: 3D shapes and volume

Grade 7, 8 and 9

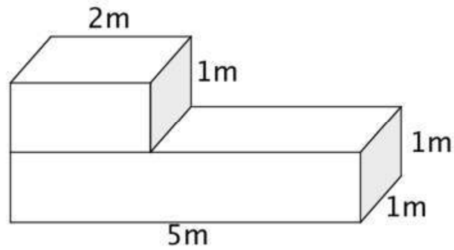
1. a) Find the volume of the rectangular prism in mm^3
b) Convert your answer to a volume in cm^3
c) Find the surface area of the prism



2. a) Find the volume of the cube in cm^3
b) Find the surface area of the cube in cm^2



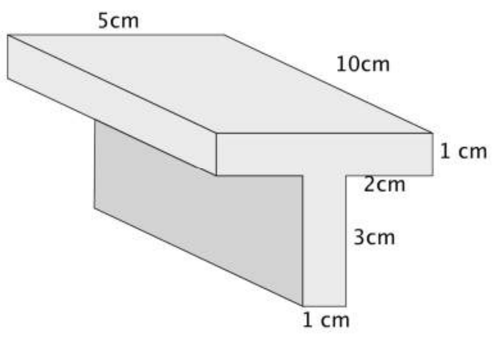
3. I have two boxes that are both rectangular prisms. I stick them together as shown alongside.
Calculate the surface area of this object.



4. A swimming pool is shaped as a rectangular prism. It is 3 m long, 2 m wide and 1 m deep.
a) Find the volume of water (in m^3) in the swimming pool if it is filled to the top.
b) How many kilolitres of water does the pool contain?

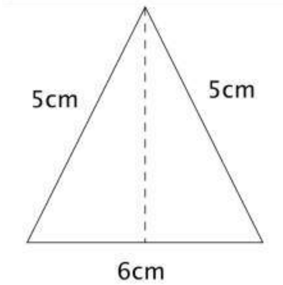
5. a) A cube has a volume of 25,625 cm^3. What is the length of the side of the cube?
b) A cube has side length of 5 cm. What is the volume of the cube?

6. This solid shown below is made up of two rectangular prisms that have been joined.
a) What is the volume of the solid?
b) What is the surface area of the solid?



Grade 8 and 9

7. a) Find the height of the triangle shown alongside.
   b) A triangular prism has this triangle as a base and is 20 cm high.
      i) What is the volume of the triangular prism?
      ii) What is the surface area of the triangular prism?

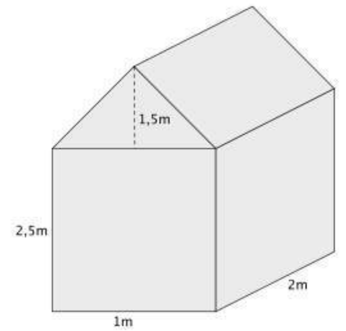


8. I make a large wooden cube with a side length of 4 m. I want to paint the outside of the cube, but I will not paint the bottom face of the cube that rests on the ground.
   a) Calculate the total area that I will paint
   b) I estimate that I need 1 litre of paint for every 2 m^2 of wood I paint. How much paint will I need to do the job?

9. The base of a triangular prism is an equilateral triangle with all sides = 6 cm. The height of the prism is 12 cm. Calculate
   a) the volume of the prism
   b) the surface area of the prism

10. A shipping container has the shape of a rectangular prism and is 5,8m long, 2,3 m wide and 2,3 m high.
    a) What is the total volume of the shipping container?
    b) The cost of shipping goods from SA to England is R40 000 per container. I have about 90 m^3 of goods I need to ship in containers. How much is that likely to cost me?

11. A wooden hut is made from a rectangular prism and a triangular prism.
    You want to paint the hut. You will not paint the face that is on the ground. What is the surface area of the hut that you will paint?

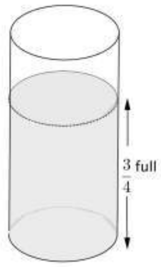


Grade 9

12. a) Calculate the volume of a cylinder with diameter 2 m and height 5 m.
    b) Calculate the surface area of a cylinder with diameter 2 m and height 5 m.

13. a) A box is 5 m long, 2 m wide and 3 m high. What is its volume?
    b) If the length of each of the dimensions of the box is doubled, what will the volume of the new box be?

14. I have a drinking glass that is the shape of a cylinder with radius 4 cm and height 10 cm. I pour water into the glass so that the glass is ¾ full.
    a) What is the volume of water in the glass?
    b) I add 2 ice cubes to the water in the glass. Each ice cube has side lengths of 2 cm. Will this cause the water in the glass to overflow?



Unit 5.1: Collect, organise and summarise data

Data handling is a way to collect and organise information about any topic or issue e.g. social, academic, sporting, health. We can 'summarise' data by finding different kinds of averages and the range of the data. We can represent the data in different ways eg graphs and tables and then interpret or analyse to provide 'evidence' for a point of view or a burning issue.

Collect data:

- ✓ What issue do you want to know about?
- ✓ Choose a **source** for the collection of data e.g. a group of people or books, magazines and newspapers.
- ✓ **Population** is ALL of the people in your source group. What is the population of the source of your data?
- ✓ A **sample** is a smaller part of the population. Sometimes the population is too big to question everyone, so you can use a big enough group to represent the whole population.
- ✓ Design a **questionnaire** to collect the data. Use the following types of questions:
  - Yes/No - the answer can only be yes or no
  - Multiple choice - give people about 3 – 5 choices
  - Rating questions - where the answer is a rating (Rate the condition of the sports grounds at school: **1** – poor **2** – need improving **3** – good enough **4** – excellent
  - Open ended - leave a space to write the exact answer that the person gives.

Worked example:

- ✓ Fikile wants to find out which sport is most popular at his school, Lethlabang Secondary.
- ✓ Source: The learners at his school.
- ✓ Population: 1 456 learners (all the learners at his school)
- a) Sample: Fikile cannot question 1 456 learners in a week. He chooses a sample of 20 learners from each grade. This is a total of 100 learners from Grade 8 to Grade 12.
- ✓ Questionnaire:

Letlhlabang Secondary
Survey by Fikile Masondo
Please circle your answers.

1. *What grade are you in?*
   8 9 10 11 12
2. *Do you play sport?*
   Yes No
3. *How many times a week do you play sport?*
   1 2-3 More than 3
4. *What sports do you play (list all)?*
   ____
   ____

Grade 7, 8 and 9

Organise data:

✓ The collected data needs to be organised or sorted.

Tally table:

✓ Data can be organised into a tally table.
✓ Each tally represents one piece of data. Tallies are grouped into 5s for easy counting. This is 5 tallies: ~~IIII~~
✓ The tallies can be added easily to give you the **frequency** of that data.
✓ If there is a lot of numeric data, it can be **grouped** into intervals so that it is easier to sort.

Stem and leaf diagram:

✓ Data can also be organised into a stem and leaf diagram.
✓ The largest place value of the numbers in the data list is written in the 'stem' and the remaining place values of each number are written in the 'leaf'.

Worked example:

The following marks (out of 50) were recorded for a class of 16 learners in a Mathematics test:

Only one number that is between 0 and 10, so use one tally

15	27	42	27	3	27	31	20
35	21	40	16	16	18	43	22

This data is grouped into intervals of 10 to make it easy to sort:

Mark (x) (out of 50)	Tally	Frequency
$0 \leq x < 10$	I	1
$10 \leq x < 20$	IIII	4
$20 \leq x < 30$	~~IIII~~ I	6
$30 \leq x < 40$	I I	2
$40 \leq x \leq 50$	I I I	3
	TOTAL	16

Three numbers between 40 and 50, so use 3 tallies

Stem and leaf diagram:

stem	leaf
0	3
1	5 6 6 8
2	0 1 2 7 7 7
3	1 5
4	0 2 3
5	

This represents 3, which has no tens value.

This represents 40, 42 and 43

Grade 7, 8 and 9

Summarise data:

Measures of central tendency:

We can use the organised data to find the different kinds of 'middle' or 'average' of the data.

- ✓ Mean: The sum of all the data values divided by the number of data values.
- ✓ Median: The middle value of the data when it is ordered from smallest to largest. When there is an even number of values in the data set, the median lies halfway between the middle two values. Add these two values and divide by 2.
- ✓ Mode: The data value that occurs most often in the data set.

Measures of spread (dispersion):

Range

The range tells you whether a data set is spread out or not.

Range = highest value – lowest value

Extremes or Outliers

Data values that are much smaller or much bigger than the other values in the set are called extremes or outliers.

If these values affect the mean and **skew** the data too much, it is better to use the median as the 'middle' of the data.

Worked example: *(continued)*

The marks for the class of 16 learners for a Mathematics test are ordered from smallest to largest:

3; 15; 16; 16; 18; 20; 21; 22; 27; 27; 27; 31; 35; 40; 42; 43

Mean:

$$\underbrace{\frac{3+15+16+16+18+20+21+22+27+27+27+31+35+40+42+43}{16}}$$

Sum of data

Number of data values

$= \frac{403}{16} = 25{,}2$ (to one decimal place)

Median: 3; 15; 16; 16; 18; 20; 21; **22; 27;** 27; 27; 31; 35; 40; 42; 43

There are two values in the middle: 22 and 27 so

median = $\frac{22+27}{2} = 24{,}5$

Mode: 27

Range = highest value – lowest value
= 43 – 3 = 40

The lowest value, 3, in the ordered data is much smaller than the next value, 15. It lies 'outside' the rest of the data and is called an 'outlier'.

Grade 7, 8 and 9

Worked example 2:

The heights of a group of Grade 9 boys are listed (in cm):

163	182	158	162	161	165	156	154	160	159

Sum: 163 + 182 + 158 + 162 + 161 + 165 + 154 + 160 + 159 = 1 464

Mean: $\frac{1\,464}{9}$ = 162,67 cm

182 cm is an **outlier**. It increases the spread of the data and it increases the value of the mean, so that it is not a true 'average' of the data.

154	158	159	160	**161**	162	163	165	182

The median of 161 cm is a better 'average' of the data.

Questions

Unit 5.1: Collect, organise and summarise data

Grade 7, 8 and 9

1. Find the mean, median, mode and range of the following sets of data:

a)

93	47	47	47	10	83	14	44	27	91
44	84	72	60	47	84	23	15	38	87

b)

60	29	26	86	29	79	31	25	95	84
20	84	36	79	13	69	84	26	89	24

c)

59	28	73	45	27	69	79	82	10	45	36	42	97	47	73

d)

23	71	23	71	59	59	16	19	8	8	18	76	52	57	57

2. A group of Grade 7 learners investigate the number of brownies sold at the tuckshop over a period of 15 consecutive days. The results are as follows:

24 22 18 26 29 31 32 20 19 22 31 27 26 25 22

a) Draw a stem and leaf diagram to organise the data.

b) Using your diagram to help you, determine the:

i) Range ii) Median

iii) Mean iv) Mode

c) Using these statistics, describe the brownie sales.

3. Anita collected data from a sample of Grade 7 learners about how far they live from the nearest grocery store. Below are the results. The values are in kilometres, correct to one decimal figure.

0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6
0,6	0,7	0,7	0,7	0,8	0,8	0,8	0,9	0,9	0,9	1	1	1	1,5	1,5
2	2	2	2	2,5	2,5	3	3	3	3,5	3,5	4	4	4	4,5
5	5	6	6	7	7	8	8	9	10	10	15	20	23	30

a) Copy and complete the table to indicate how many of the values appear in each of the given intervals.

b) What percentage of the learners live less than 1 km from a grocery store? Round off to one decimal place.

Interval	Frequency
less than 1,0 km	
1,0 – 5,9 km	
6,0 – 9,9 km	
10 km or further	

4. The table shows the body weights (in kg) of athletes competing in a tournament.

55,2	56,1	58,4	59,3	60,6	61,2	61,7	63,4
63,2	64,2	65,9	66,5	66,7	67,3	67,8	68,0
70,5	72,9	73,4	74,1	74,8	75,9	76,7	78,7

a) Group the weights into 5 kg intervals. List the intervals.

b) Use a table to show the frequency of each class interval. It is useful to fill in the tallies first and then count up the frequencies, so that you don't leave any data items out.

Body weights of athletes	Tally	Frequency

c) In which intervals are the highest numbers of athletes?

5. Below is a stem-and-leaf plot showing the mass of 6-week-old chickens on a farm. (Key: 35| 4 means 354 g)

a) What is the mass of the lightest 6-week-old chicken on the farm?
b) What is the mass of the heaviest 6-week-old chicken on the farm?
c) What is the median mass of the 6-week-old chickens on the farm?

34	0 4
35	4 8 8
36	0 1 6 8
37	1 3 5 8 8 8 9
38	2 4 9
39	0 3 4 4 5 6 9
40	0 3 7
41	1

6. Here are the Mathematics test results, out of 30, of a small class of 21 learners.

15	7	11	7	13	4	8	9	3	7	25
7	6	10	8	9	23	19	7	5	7	

Bongile scored 9 out of 30 in the test, which is poor. Can he claim that his mark in in the top half of the class? Explain your answer well.

7. A journalist investigated the price of white bread at different stores in two large cities. The prices in cents at 10 different shops in each city are given below.

City A	927	885	937	889	861	904	899	888	839	880
City B	890	872	908	910	942	924	900	872	933	948

a) If you just look at the data, do you think one can say that white bread is cheaper in the one city than in the other? Look carefully, and give reasons for your answer.

b) Calculate the mean price of white bread for the sample in each of the two cities.

c) Find the median bread price in the sample for each of the two cities.

8. The following data is collected for a project. It represents the number of times nine different people have accessed their Facebook page in the past week:

5; 8; 8; 10; 15; 17; 18; 23; 59.

Which measure of central tendency (the mode, mean or median) will best represent the 'average' of the data? Explain your answer.

Unit 5.2: Represent data with graphs

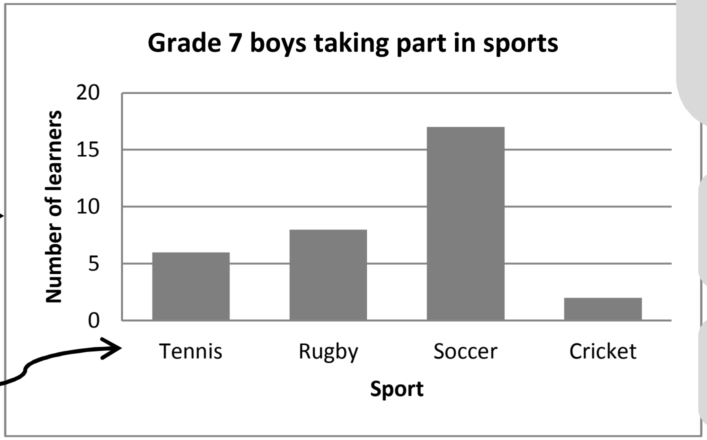
Represent data:

Graphs represent data well because they give a picture of the data that is easy to interpret.

A. Bar graphs:

Bar graphs are used for discrete data (eg number of learners) or for data in categories (eg colours of cars, types of sport)

Frequency (number in the data group) on the vertical axis



Group names on the horizontal axis.

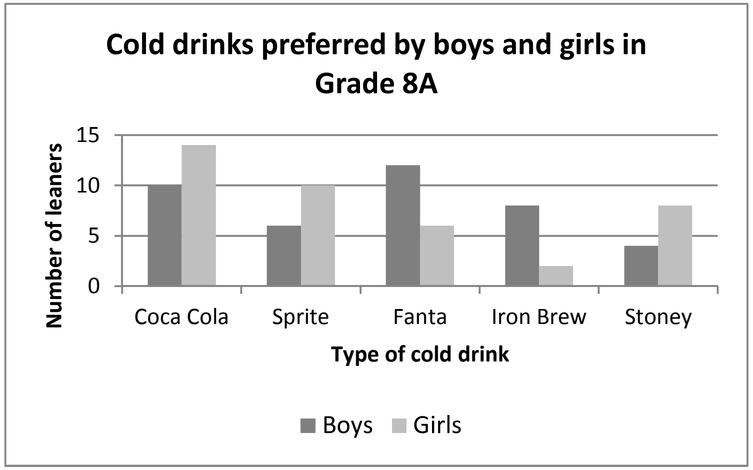
<u>All graphs must have</u>:

- ✓ Heading – describe what the data shows.
- ✓ Labels and units on both axes.
- ✓ A key if needed to explain categories.
- ✓ An accurate scale on vertical axis.

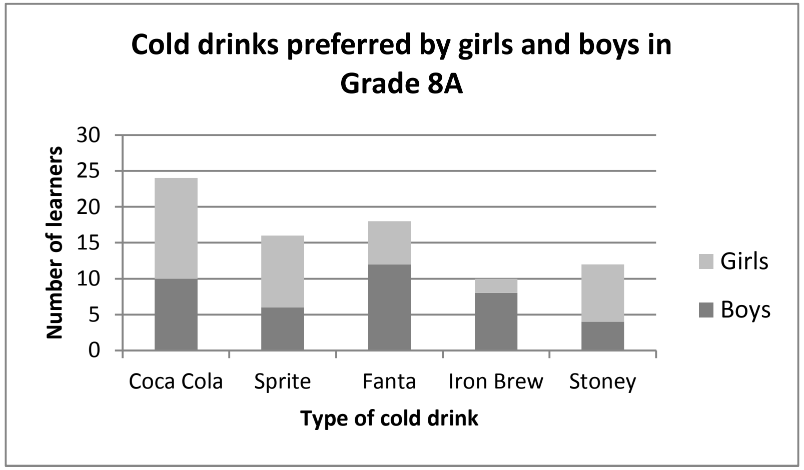
The bars are all the same width.

There are equal spaces between the bars.

<u>Double bar graph</u>: the data for each category is



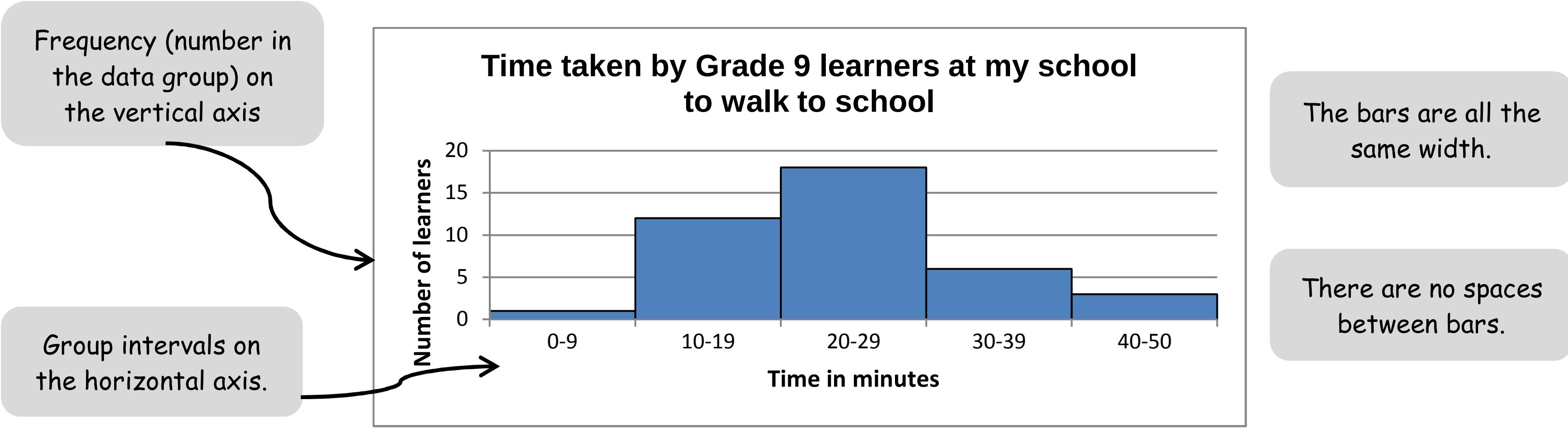
<u>Stacked bar graph</u> (the data from the double bar graph separated into two parts (eg boys and girls) is stacked on top of each other):



Grade 7, 8 and 9

B. Histograms:

Histograms are used for continuous data which in groups or intervals. This data is not restricted to whole numbers e.g. temperature of heating water; time taken to walk from school.



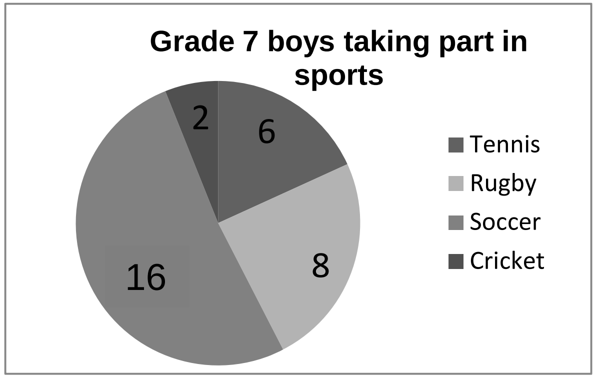
C. Pie charts:

Pie charts can be used for data in categories. You can compare the size of each category with the total of all the data.

<u>Steps to draw a pie chart</u>:

- ✓ Organise the data into a frequency table.
- ✓ Find the total of the frequency column.
- ✓ Divide the frequency of the category by the total frequency to find the fraction of the pie chart.
- ✓ Divide the pie into equal sectors and then shade them to show each category.
- ✓ Label each sector or use colours and a key to label the categories.

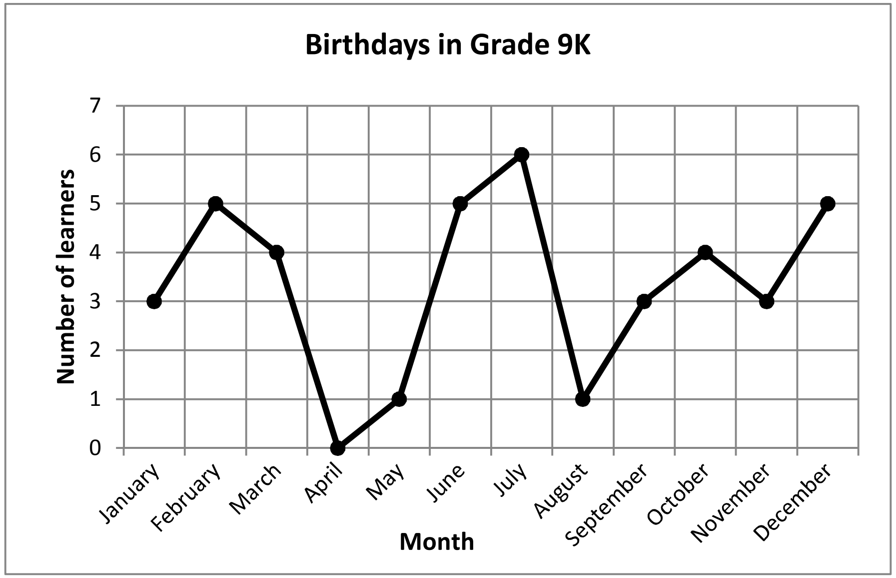
Sport	Frequency	
Tennis	6	$\frac{6}{32} = \frac{3}{16}$
Rugby	8	$\frac{8}{32} = \frac{4}{16}$
Soccer	16	$\frac{16}{32} = \frac{8}{16}$
Cricket	2	$\frac{2}{32} = \frac{1}{16}$
Total	32	



D. Broken line graphs: Grade 8 & 9

A broken line graph is used for data in categories where the categories are related to each other or follow on from each other. For example, the categories might be consecutive times, days, months or years.

Note: In these graphs the line does not indicate continuity between the categories. Its purpose is to highlight the trend of the data.

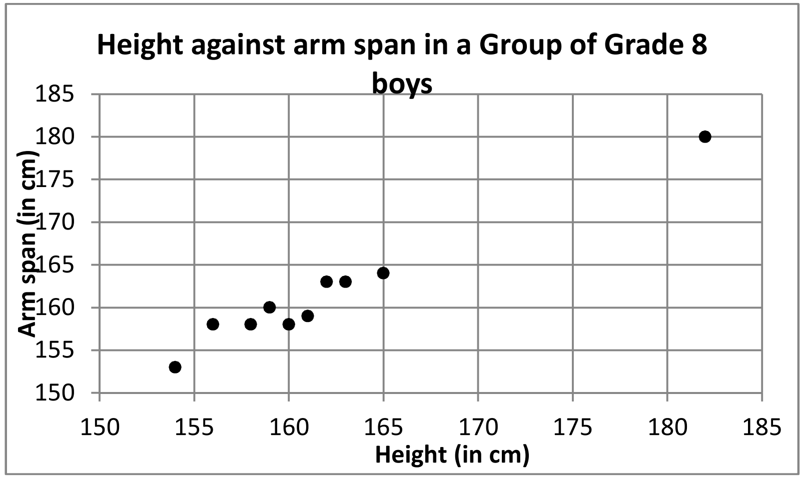


E. Scatter plots: Grade 9

Scatter plots are used to graph data points that have two values associated with them. Data values have two independent measurements. e.g. Grade 8 boys who measure their arm span and their height.

Steps to draw a scatter plot:

- ✓ Decide which value to plot on the x axis and which value to plot on the y axis.
- ✓ Plot each data point as an ordered pair on the axes.
- ✓ **Do not join the points** in a scatter plot.

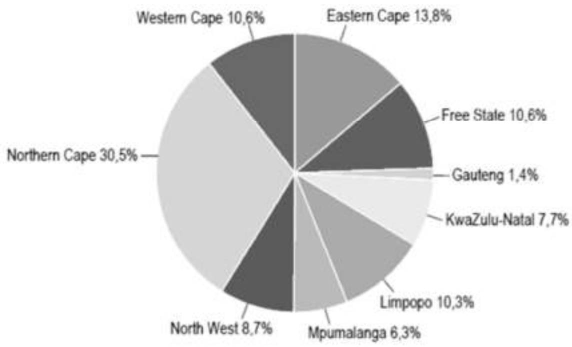


Questions

Unit 5.2: Represent data with graphs

Grade 7, 8 and 9

1. The pie chart here shows the land area of each province as a percentage of the land area of the country.



Answer the following questions:

a) Which province has the biggest area?

b) What size (in °) is the sector representing the Free State.

c) The total area of South Africa is $1\ 213\ 090\ \text{km}^2$. How big (in km^2) is Limpopo?

http://www.statssa.gov.za/publications/P03014/P030142011.pdf

2. The table shows the population by province over 3 census years

a) Using the data from 2001 and 2011 (*do not use the data from 1996),* draw a double bar graph.

b) Use your graph to determine which province had the largest growth in population between 2001 and 2011.

Table 3.1: Total population by province, Censuses 1996, 2001 and 2011

Province	Census 1996	Census 2001	Census 2011
Western Cape	3 956 875	4 524 335	5 822 734
Eastern Cape	6 147 244	6 278 651	6 562 053
Northern Cape	1 011 864	991 919	1 145 861
Free State	2 633 504	2 706 775	2 745 590
KwaZulu-Natal	8 572 302	9 584 129	10 267 300
North West	2 727 223	2 984 098	3 509 953
Gauteng	7 834 125	9 388 854	12 272 263
Mpumalanga	3 123 869	3 365 554	4 039 939
Limpopo	4 576 566	4 995 462	5 404 868
South Africa	**40 583 573**	**44 819 778**	**51 770 560**

Censuses 1996 and 2001 have been aligned to 2011 municipal boundaries

(Source:http://www.statssa.gov.za/publications/P03014/P030142011.pdf).

3. In a Natural Sciences class, learners planted beans and measured the heights of the bean plants after two months. Here is the data they collected (in cm):

34	65	72	42	37	29	78	43	79	91	43	45	28	42	79
34	92	87	40	43	43	78	82	47	85	43	32	86	76	

a) Copy and complete the frequency table. b) Draw a histogram of this data.

Height of bean plants (cm)	Tally	Frequency
20 – 29		
30 – 39		
40 – 49		
50 – 59		
60 – 69		
70 – 79		
80 – 89		
90 – 99		
Total		

4. a) Draw a histogram to represent the data in the table below.
Group the data in intervals of 0,5 kg.

Birth weights (kg) of 28 babies at a clinic

3,3	1,34	2,88	2,54	1,87	2,06	2,72
1,89	0,85	1,99	2,43	1,66	2,45	1,62
1,91	1,20	2,45	1,38	0,9	2,65	2,88
1,75	2,11	3,2	1,74	0,6	3,1	1,86

b) Calculate the mean and the median of the data.
c) Records from the whole country show that the birth weights of babies ranges from 0,5 kg to 4,5 kg, and the mean birth weight is 3,18 kg. Use the graph and the mean and median to write a short report on the data from the clinic.

Grade 8 and 9

5. The table shows the income of Pam's small business and Luthando's small business over 6 months.

Month	January	February	March	April	May	June
Pam's income (R)	12 000	12 000	9 000	6 000	7 000	9 000
Luthando's income (R)	6 000	7 000	8 000	8 000	9 000	9 000

a) Draw a broken-line graph showing Pam's income.
b) Draw a broken-line graph showing Luthando's income.
c) Whose income seems to be increasing steadily per month?

Grade 9

6. The table below shows the number of hours learners in 9E studied for a Mathematics test and the mark they achieved.

Hours studied	3	1	5	1	0	7	4	5	3	4	5	2	4	1	6
Mark (%)	56	35	68	20	32	73	57	63	46	41	71	52	43	41	90

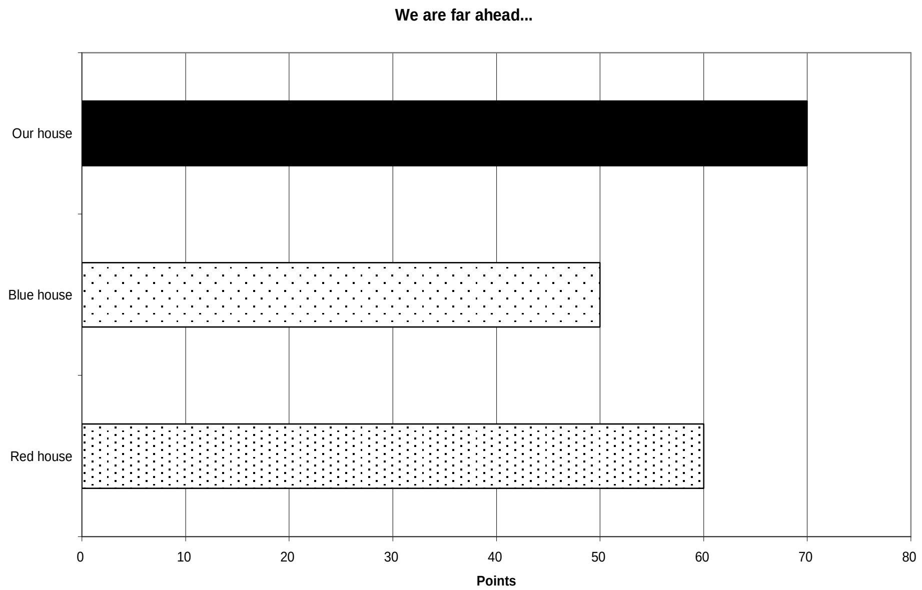
a) Represent this data in a scatter plot.
b) Do you think there is a positive relationship between hours studied and mark achieved? Explain using your graph.

Unit 5.3: Bias and error in data

Sources of error and bias in data:

When you are given data and graphs to study, it is possible that the data is **biased**. Bias occurs when the person presenting the data wants the reader to reach a particular conclusion and so they emphasise what they want you to notice.

Consider the graph below. In what ways is this graph biased?



- ✓ The heading is biased - it does not say what the graph is about but rather what the author wants you to think.
- ✓ "Our house" is the top bar and it is coloured in the darkest colour making you think it's the most important.
- ✓ If you look at the data, "our house" is only ten points ahead of the "red house" so that are not that far ahead.

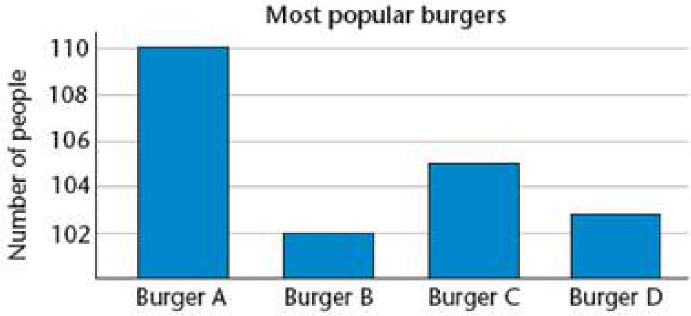
There are many ways that data can be biased.
Here are some examples to look out for:

- ✓ Scales on the axes - stretching or squashing the scale can make the graph look flatter or steeper.
- ✓ Changing the steepness of the graph can make increase or decrease look faster or slower and change the readers feeling toward the information shown.
- ✓ Look at the heading and axes headings for bias.
- ✓ Look at which information is presented first and what is presented last.
- ✓ If possible, look at the source of the data. Was the sample big enough? Was the sample random? Who was in the sample group - did they have any bias towards the research?

Grade 7, 8 & 9

Grade 7 – 9

1. Look at the bar graph below and answer the following questions:



a) Which burger is the clear favourite (most popular)?
b) The height of the bars indicate that burger A is liked by five times as many people as burger B. Is this true? Look at the vertical scale.

2. The manager of a small business is asked what monthly salaries his employees get. His answer: *The mean of the salaries is R13 731.*

a) Do you think the manager's answer is a good description of the salaries?

b) In order to have some sense of the salaries paid at the firm, which one of the following would you prefer to know: the *median,* or the *mode,* or the *range,* or the *lowest and highest* salaries?

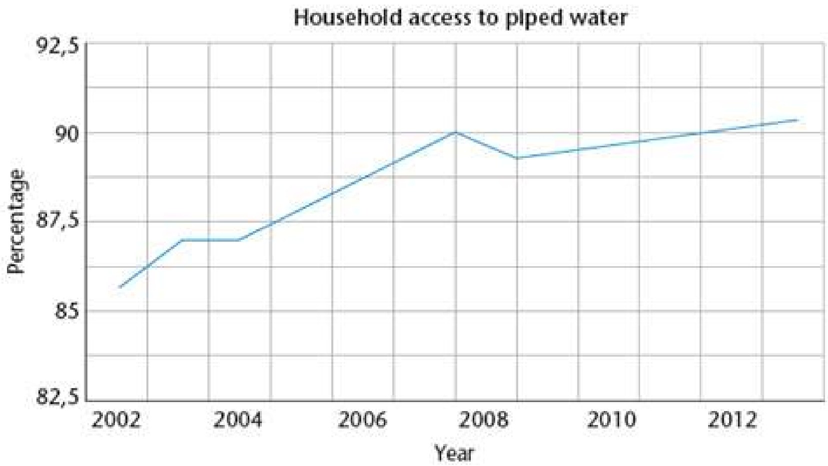
The actual monthly salaries of the 13 staff members in the small business mentioned in question 1 are given below.

R3 500	R3 500	R3 500	R3 500	R3 500
R4 200	R4 200	R4 200	R4 400	R12 000
R28 000	R44 000	R60 000		

In what ways may you be misinformed if you do not know the above figures, but only know that the mean salary is R13 731?

Grade 8 & 9

3. This graph from Statistics South Africa shows the increase in the percentage of households that had access to piped water over a ten-year period.



a) Comment on the scale used on the vertical axis. Is this a misleading graph?
b) How could you redraw the graph so that the differences on the graph are more noticeable?
c) How could you redraw the graph so that the differences on the graph are less noticeable?

Unit 10: Probability

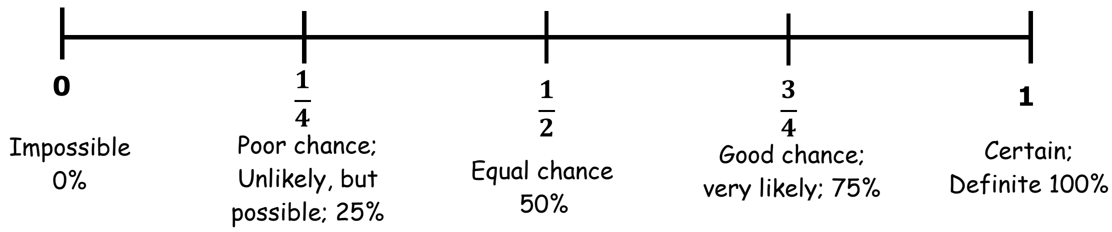
Probability is the study of how likely it is that an event will happen.

What is the chance that it will rain tomorrow?

What chance do I have of winning the Lotto?

- ✓ Probability always lies between 0 and 1, measured as a fraction or as a decimal. It can also be shown as a percentage between 0% and 100%.

  We can show probability on a scale:



- ✓ We can work out the **probability** using the formula: $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
- ✓ We can show probability as a common fraction, a decimal fraction or a percentage eg A probability of 5 out of 8 can be written as

  $\frac{5}{8}$ or as 0,625 or as 62,5%

Examples:
If a dice is rolled, the probability of getting a 5 is 1 out of 6.
We can write this as P(5) = $\frac{1}{6}$ or as P(5) = 0,167 or as P(5) = 16,7%.

Probability questions sometimes refer to dice, coins or cards. Make sure you know that:

- ✓ When throwing a dice, there are 6 possible outcomes (1; 2; 3; 4; 5; 6)
- ✓ A coin has 2 sides, heads and tails

Cards:

- ✓ There are 52 cards in a pack of cards.
- ✓ A pack of cards has 4 suits with 13 card values in each suit. So there are 4 cards of each value (i.e. there are 4 twos, threes, fours etc in a pack)
- ✓ The four suits are diamonds (red), spades (black), hearts (red) and clubs (black).
- ✓ The card values in each suit are 2; 3; 4; 5; 6; 7; 8; 9; 10; Jack; Queen; King; Ace.

Grade 7, 8 & 9

Relative frequency:

If you flip a coin and record the result:
- ✓ The possible **outcomes** are H (heads) or T (tails).
- ✓ There are two possible outcomes. Each has a 50% chance of happening.
- ✓ We say that there is a **theoretical probability** of $\frac{1}{2}$ for each outcome.

But, if you actually flip the coin 50 times, you may not get heads (the outcome you want to record) exactly 25 times. The actual number of times the coin lands on heads divided by the number of throws is called the relative frequency.

$$\text{Relative frequency} = \frac{\text{number of favourable outcomes}}{\text{number of experiments}}$$

Worked examples:
1. Bongani flips a coin 10 times and it lands on heads 4 times.
   The relative frequency of heads for this experiment is $\frac{4}{10}$ = 0,4 or 40%.
2. Bongani flips a coin 100 times and each time, he records H for heads or T for tails. His record shows that he flipped heads 55 times.
   So the relative frequency of heads is $\frac{55}{100}$ or 0,55 or 55%.
   This is closer to 50% which is what we expected.
3. As Bongani flips the coin more times, he will get a relative frequency closer and closer to 50%.

Grade 7, 8 and 9

Probabilities for compound events:

Using probability, we can also work out the chances of two events happening.

What is the chance of rolling two dice and getting a 6 on both?

4 children win prizes at a competition open to boys and girls. What are the chances that they are all girls?

Tables

You can show the number of possible outcomes for two events happening one after the other using tables.

Worked example:

A coin is tossed two times. What is the probability of getting two heads?

		Flip 2	
		H	T
Flip 1	H	**HH**	**HT**
	T	**TH**	**TT**

The probability of two heads is 1 out of 4, or $\frac{1}{4}$

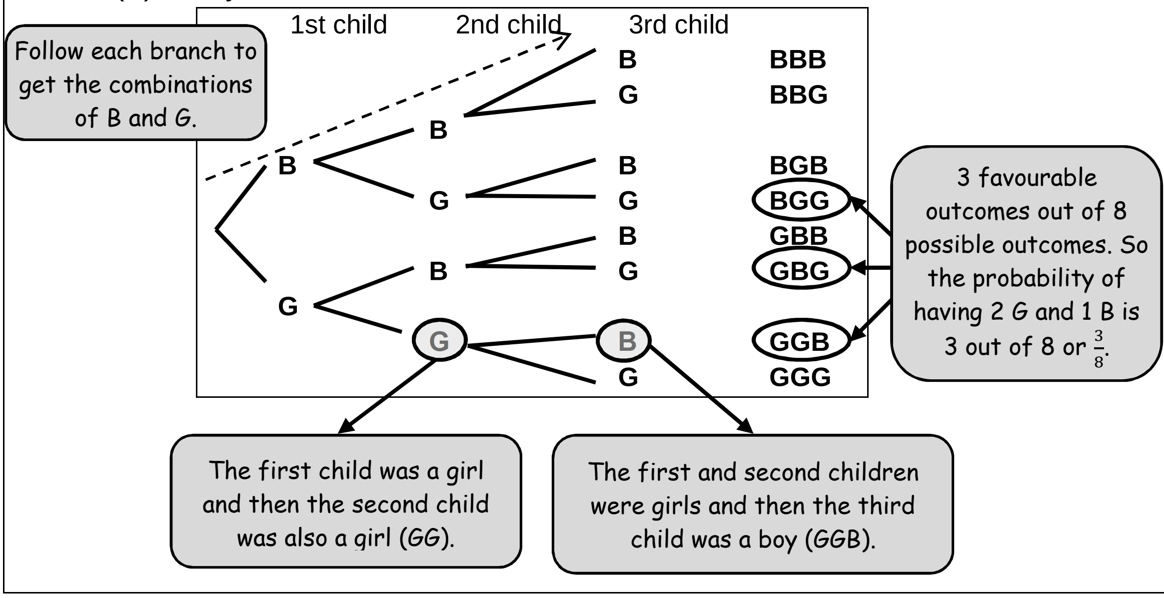
Grade 9

Tree diagrams

In a tree diagram, you can list all the possible outcomes for an experiment.

<u>Example</u>:

In a family of 3 children, what is the probability of having 2 girls (G) and 1 boy (B) in any order?



Grade 9

Questions

Unit 5.3: Probability

You may use a calculator.

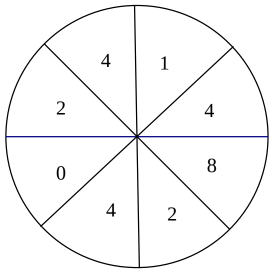
Grade 7, 8 & 9

1. If you roll a die, what is the probability of getting:
   a) a 4 b) a prime number
   c) an odd number d) an 8
   e) a factor of 6 f) a number less than 8

2. If a card is drawn randomly from a pack f 52 cards, what is the probability that it will be:
   a) a heart b) a black card
   c) a Jack, Queen or King d) a 6

3. A bag contains 10 red, 6 blue and 3 white marbles. If one marble is drawn at random, what is the probability that it will be:
   a) blue b) red
   c) not red d) white or blue
   e) green

4. A spinner is numbered as follows:
   What is the probability of getting:
   a) 2
   b) 8
   c) 4



5. A school is having a raffle to raise funds. They sell a total of 725 tickets. What is the probability of someone winning if they have bought:
   a) 5 tickets
   b) 10 tickets
   c) 2 tickets?

6. Draw a line that is 10 cm long to represent that probabilities from 0 to 1. Mark the position of the following probabilities on the line:
   a) throwing a 4 on a single roll of a dice
   b) winning the lottery if you do not buy a ticket
   c) getting heads OR tails when tossing a coin
   d) choosing a red marble from bag containing 4 red and 4 blue marbles

Grade 8 & 9

7. If you roll a dice 300 times, predict the number of times you are likely to get:
   a) a 3
   b) a multiple of 2
   c) a number more than 6

8. Susan rolls a dice and records her results. She repeats the experiment 30 times and notices that she doesn't throw any 6s. Is this what she would have expected? Explain.

Grade 9

9. Suppose you roll a dice and toss a coin one after the other.
   a) In how many ways can the dice fall?
   b) In how many ways can the coin fall?
   c) Draw a tree diagram to show all the possible combinations.
   d) What is the probability of getting:
      i) 5 and a tail
      ii) an even number and a head
      iii) a multiple of 3 and a tail

10. Two spinners are used in a game. The first spinner has the four numbers: 1; 1; 2; 3. The second spinner has the four numbers: 2; 3; 4; 5. They are both spun and the score is the sum of the numbers on the two spinners.
   a) Draw a table to determine all the possible scores in the experiment.
   b) What is/are the most likely score/s?
   c) Work out the probability of getting a score of 3.

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